Imperial College London

Lecture 2

Signals in Time Domain

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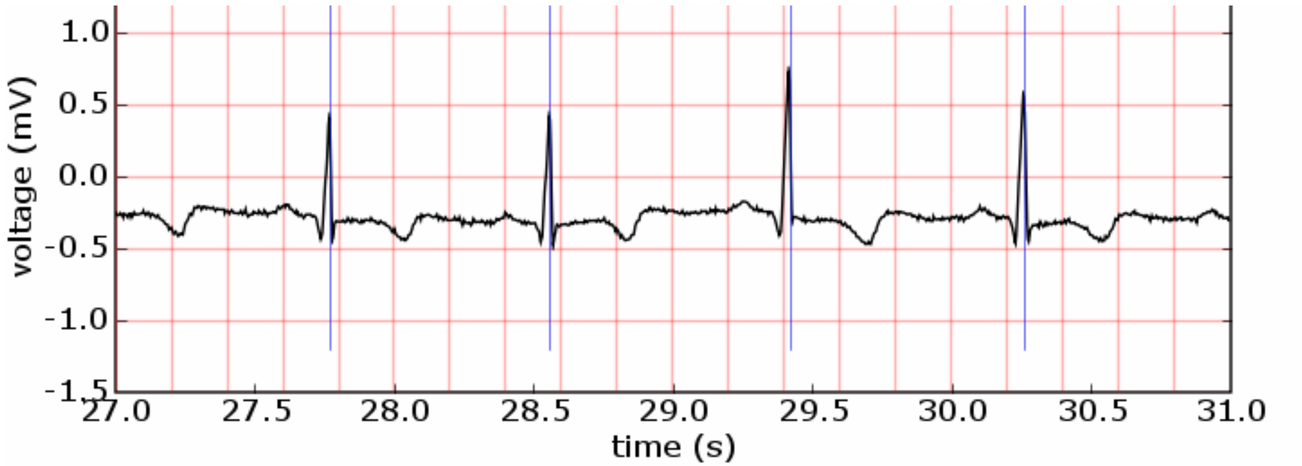
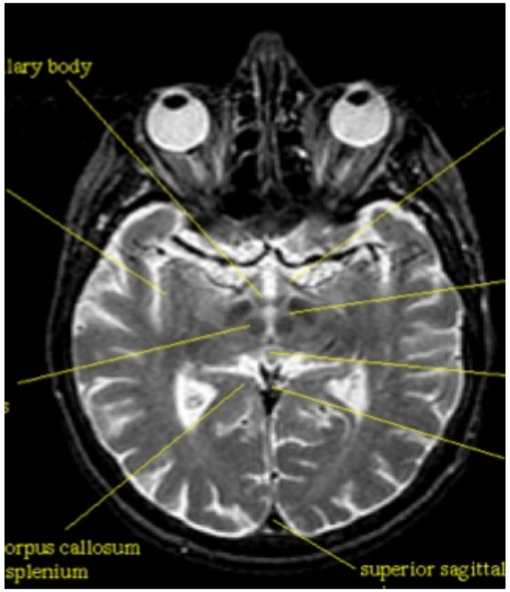
Examples of signals

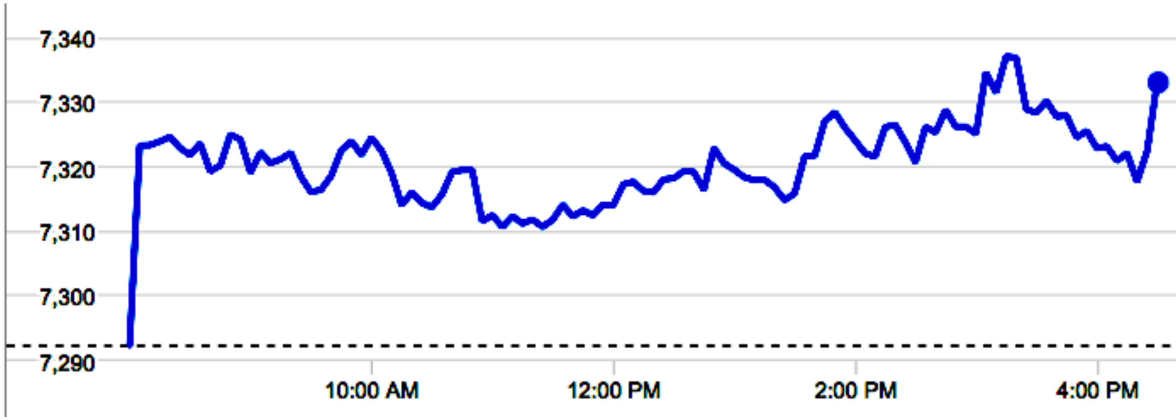
- Electrocardiogram (ECG) signal



- Magnetic Resonance Image (MRI) data as 2-dimensional signal



- FTSE 100 index in a day as signal (time series)

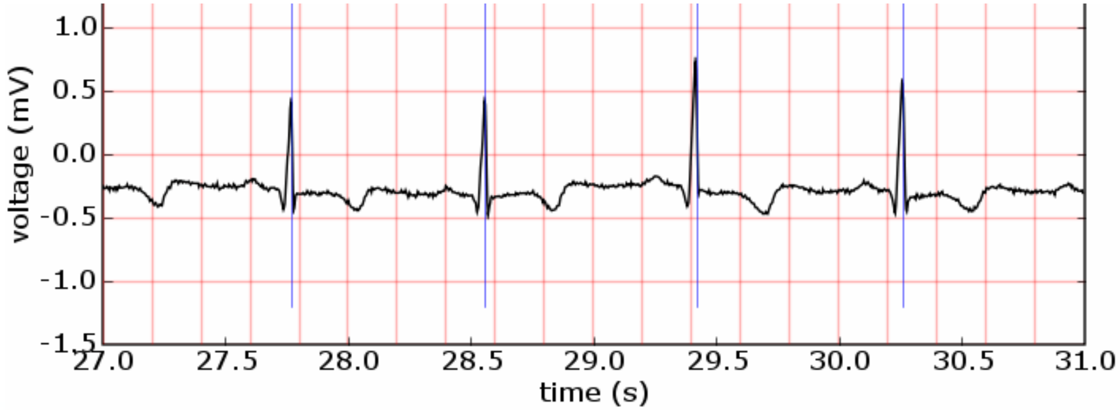


Signals Classification (1)

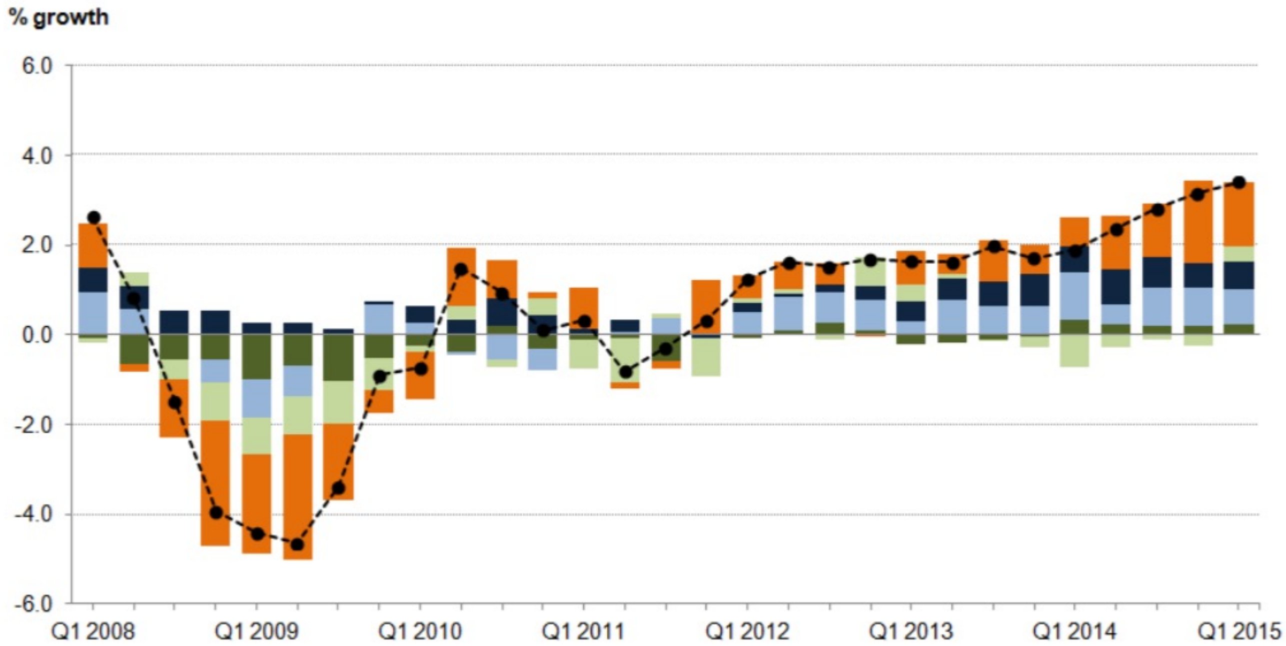
- Signals may be classified into:
  1. Continuous-time and discrete-time signals
  2. Analogue and digital signals
  3. Periodic and aperiodic signals
  4. Energy and power signals
  5. Deterministic and probabilistic signals
  6. Causal and non-causal
  7. Even and Odd signals

Signal Classification (2) – Continuous vs Discrete

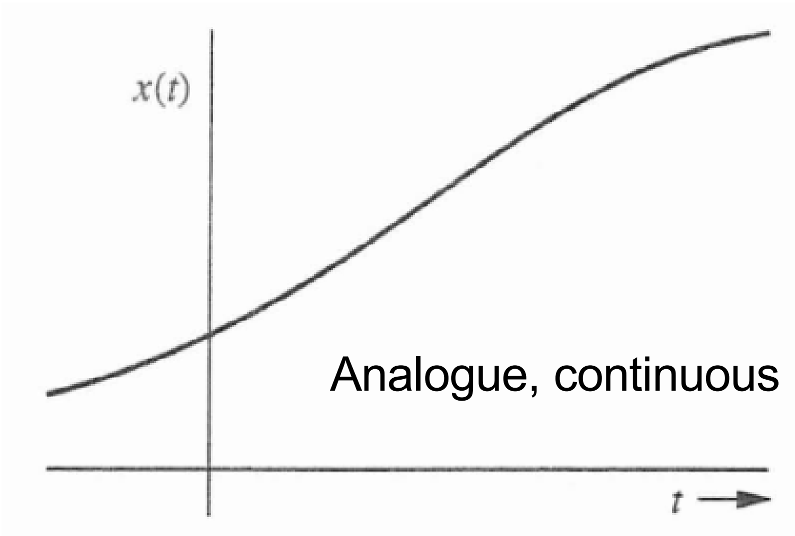
- Continuous-time (CT), e.g. ECG signal

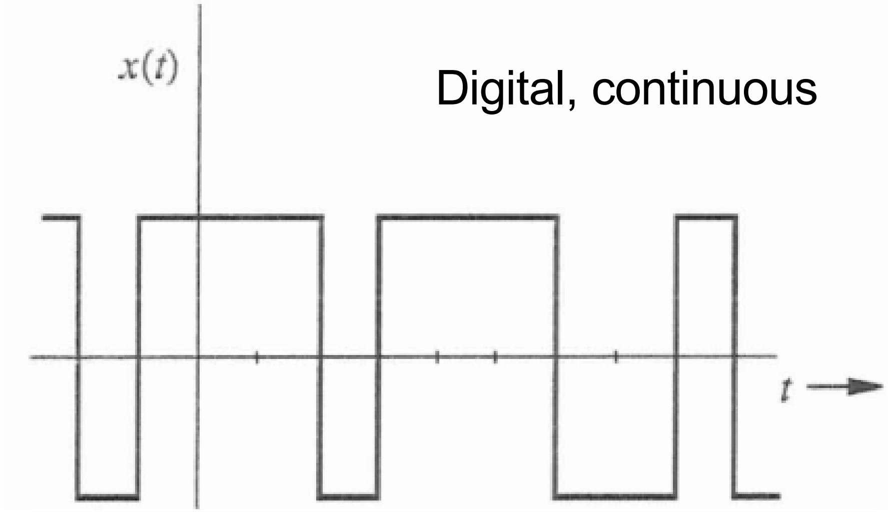


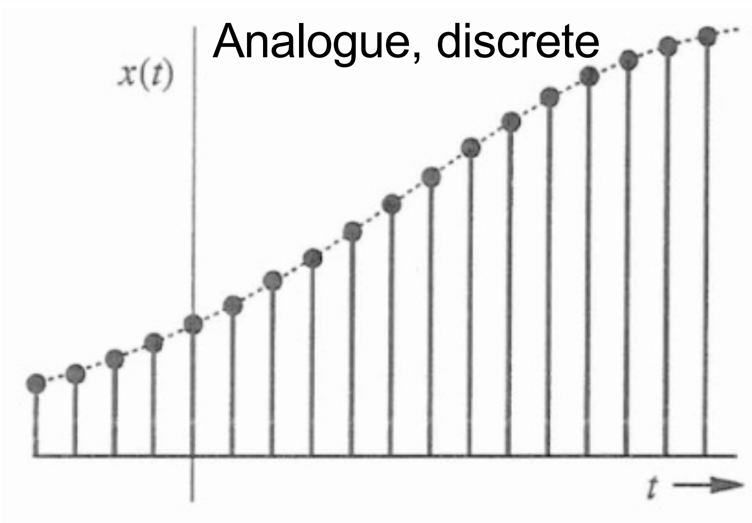
- Discrete-time (DT), e.g. UK growth rate

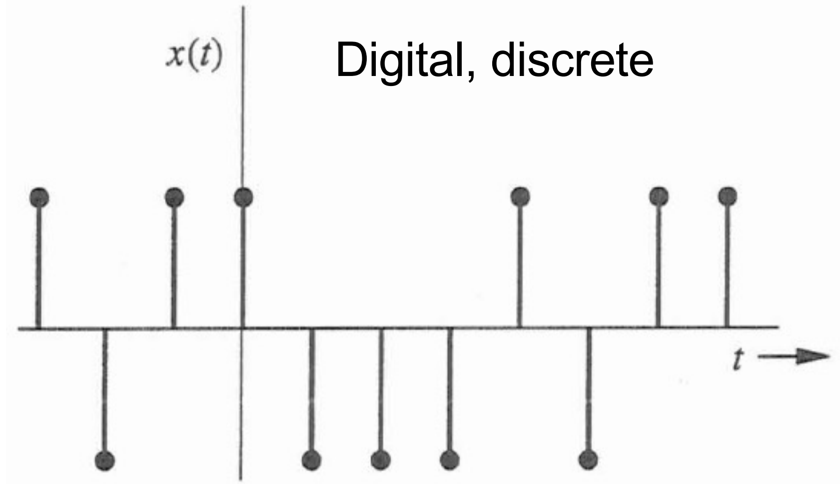


Signal Classification (3) – Analogue vs Digital







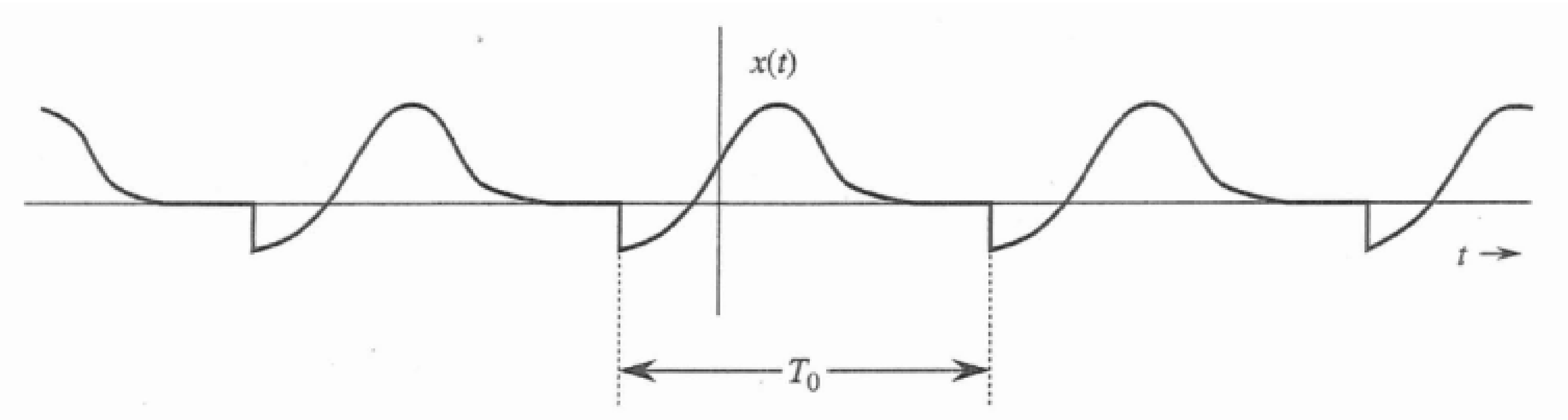


Signal Classification (4) – Periodic vs Aperiodic

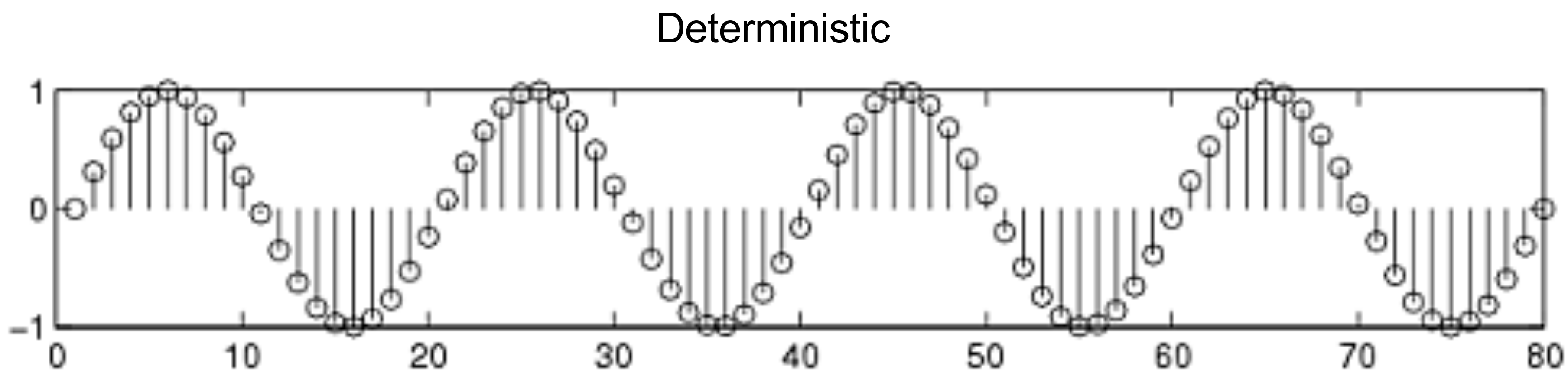
- A signal *x(t)* is said to be periodic if for some positive constant T_o

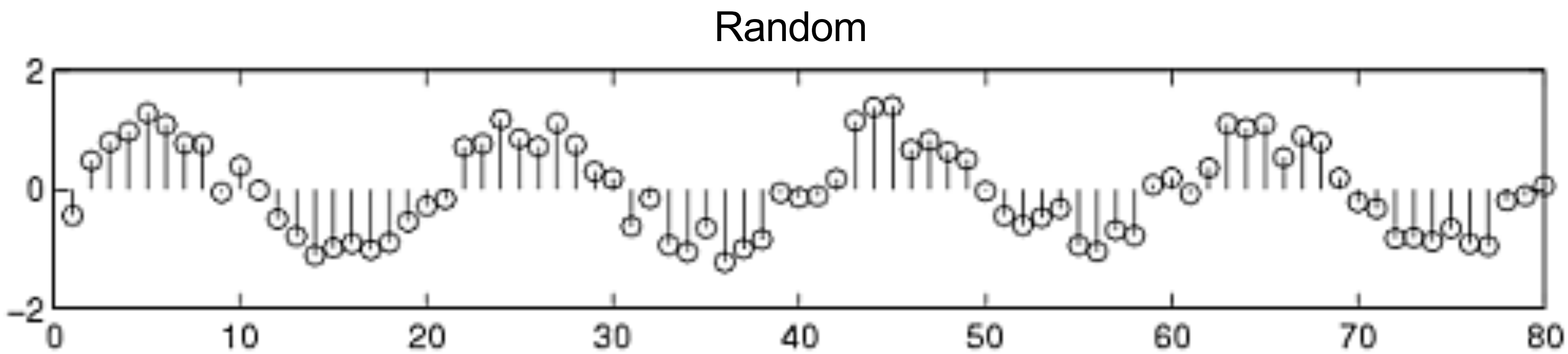
$$x(t) = x(t + T_0) \qquad \text{for all } t$$

- The smallest value of T_o that satisfies the periodicity condition of this equation is the *fundamental period* of x(t).

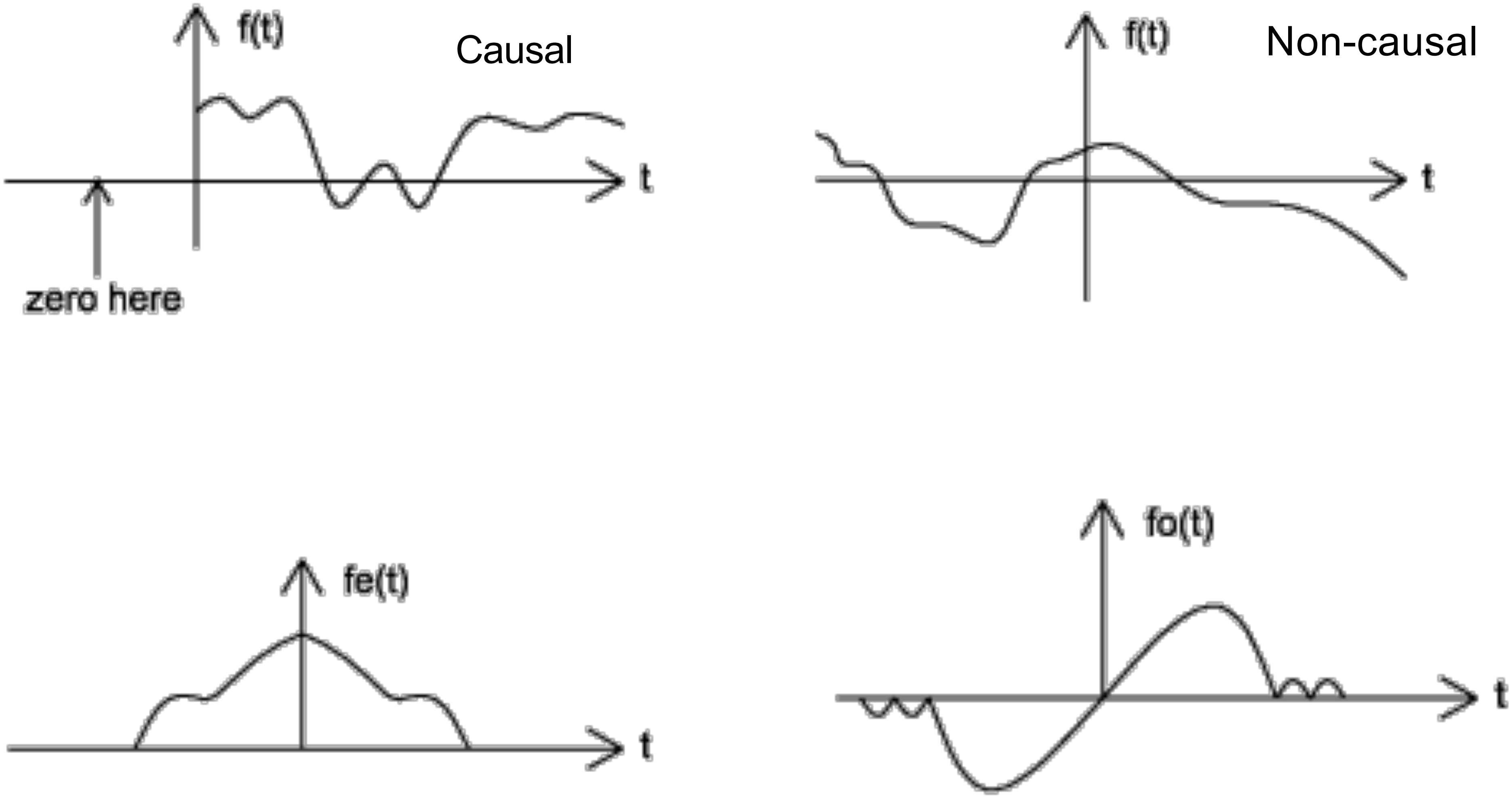


Signal Classification (5) – Deterministic vs Random

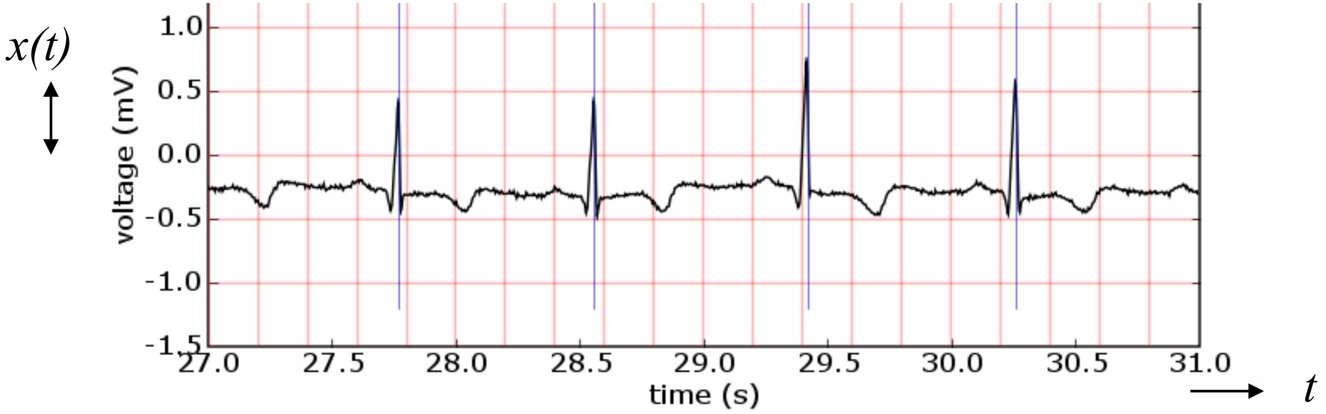




Signal Classification (6) – Causal/Non-causal, Even/Odd



Size of a Signal *x(t)* as energy



- Measured by signal energy E_x:

$$E_x = \int_{-\infty}^{\infty} x^2(t)\ dt$$

- Generalize for a complex valued signal to:

$$E_x = \int_{-\infty}^{\infty} \left| x(t) \right|^2\ dt$$

- Energy must be finite, which means

$$\text{signal amplitude} \rightarrow 0 \quad \text{as } |t| \rightarrow \infty$$

Size of a Signal *x(t)* as power

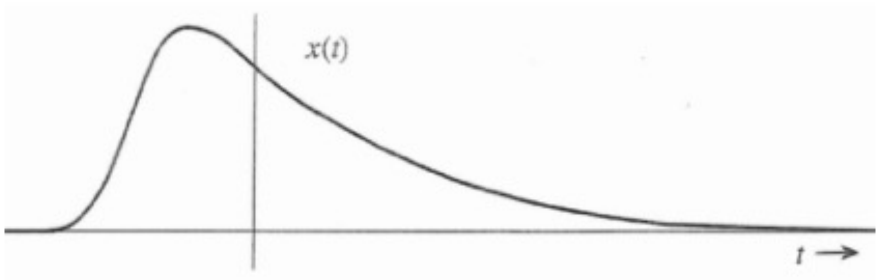
- If amplitude of *x(t)* does not $\rightarrow$ 0 when t $\rightarrow \infty$, need to measure power *Px* instead:

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

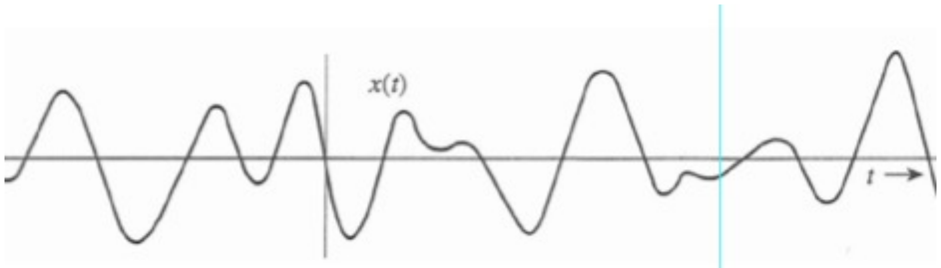
- Again, generalize for a complex valued signal to:

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt$$

- Signal with finite energy (zero power)



- Signal with finite power (infinite energy)



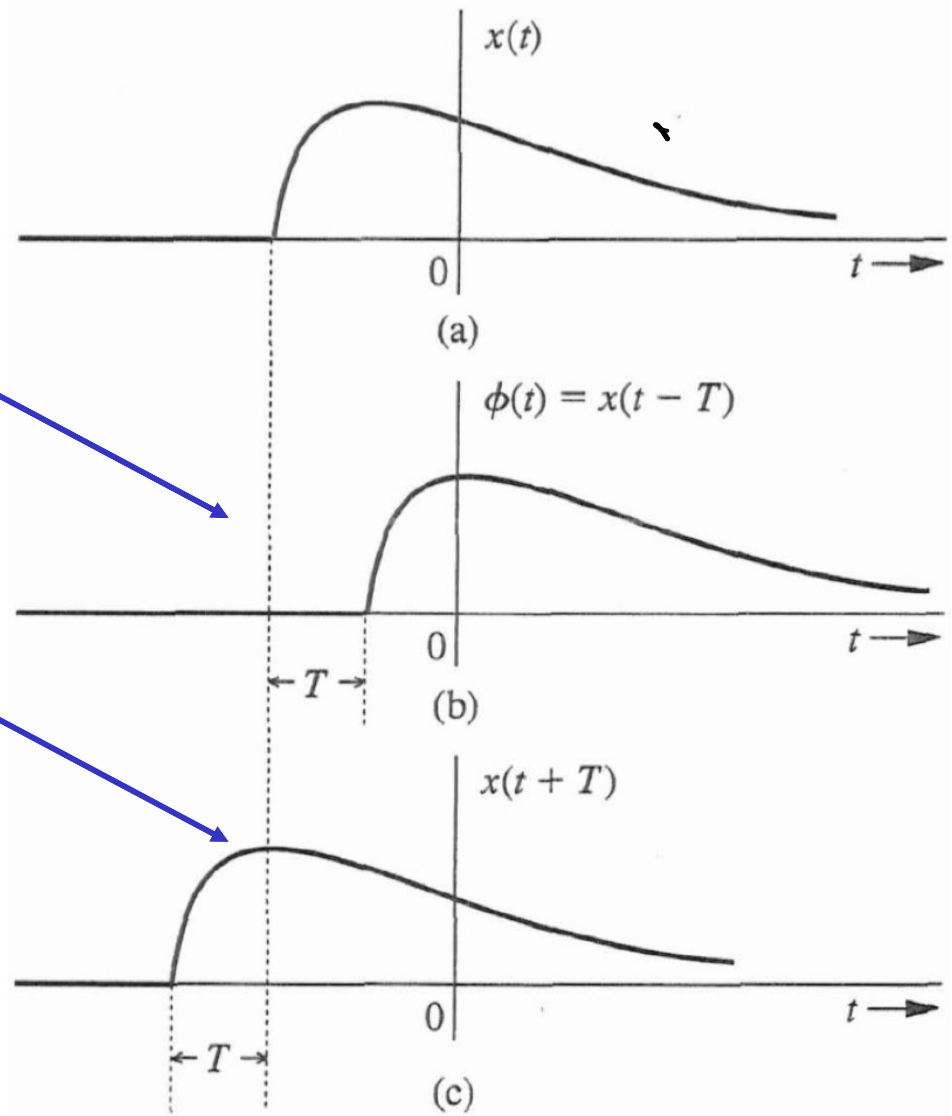
Useful Signal Operations –Time Shifting (1)

- Signal may be delayed by time T:

$$\phi(t + T) = x(t)$$

- or advanced by time T:

$$\phi(t - T) = x(t)$$



Useful Signal Operations –Time Scaling (2)

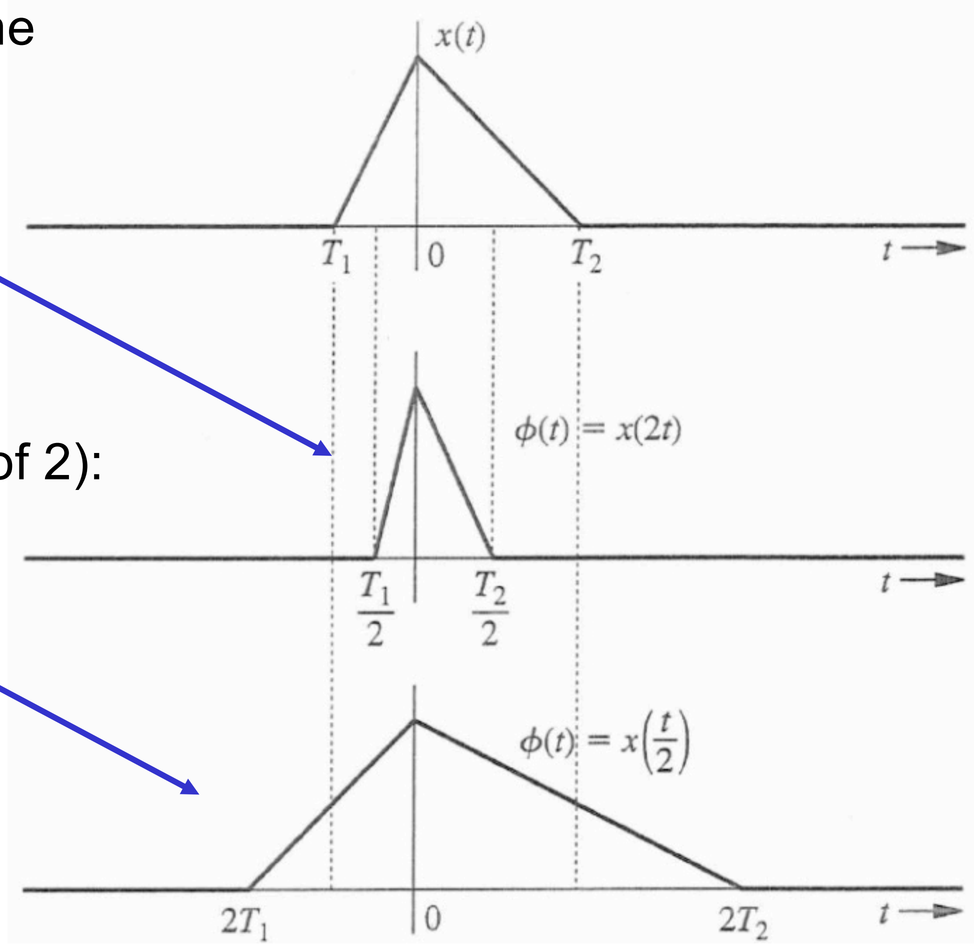
- Signal may be compressed in time (by a factor of 2):

$$\phi(t/2) = x(t)$$

- or expanded in time (by a factor of 2):

$$\phi(2t) = x(t)$$

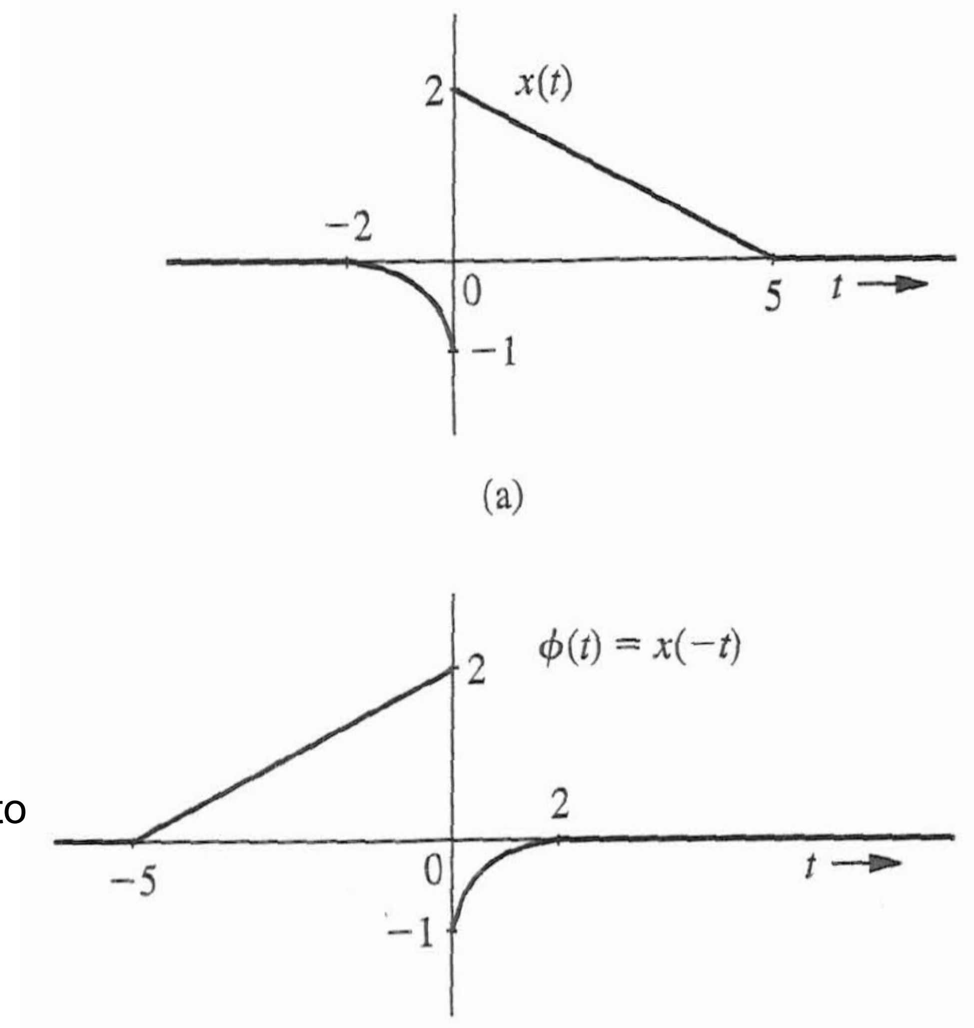
- Same as recording played back at twice and half the speed respectively



Useful Signal Operations –Time Reversal (3)

- Signal may be reflected about the vertical axis (i.e. time reversed):

$$\phi(t) = x(-t)$$



- We can combine these three operations.

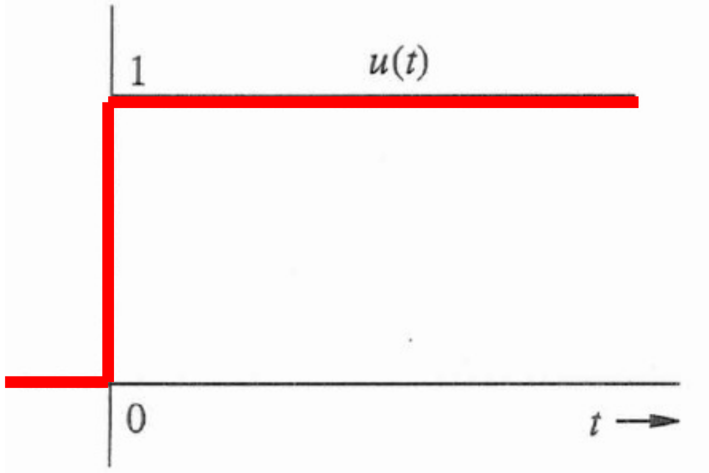
- For example, the signal x(2t - 6) can be obtained in two ways:

  1. Delay x(t) by 6 to obtain x(t - 6), and then time-compress this signal by factor 2 (replace t with 2t) to obtain x (2t - 6).
  2. Alternately, time-compress x (t) by factor 2 to obtain x (2t), then delay this signal by 3 (replace t with t - 3) to obtain x (2t - 6).

Signal Models (1) – Unit Step Function *u(t)*

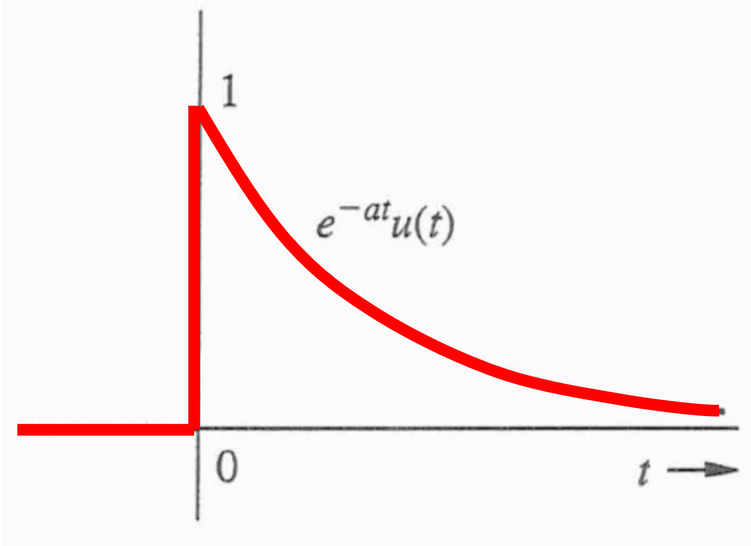
- Step function defined by:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



- Useful to describe a signal that begins at t = 0 (i.e. causal signal).
- For example, the signal e^{-at} represents an everlasting exponential that starts at t = -∞.
- The causal for of this exponential can be described as:

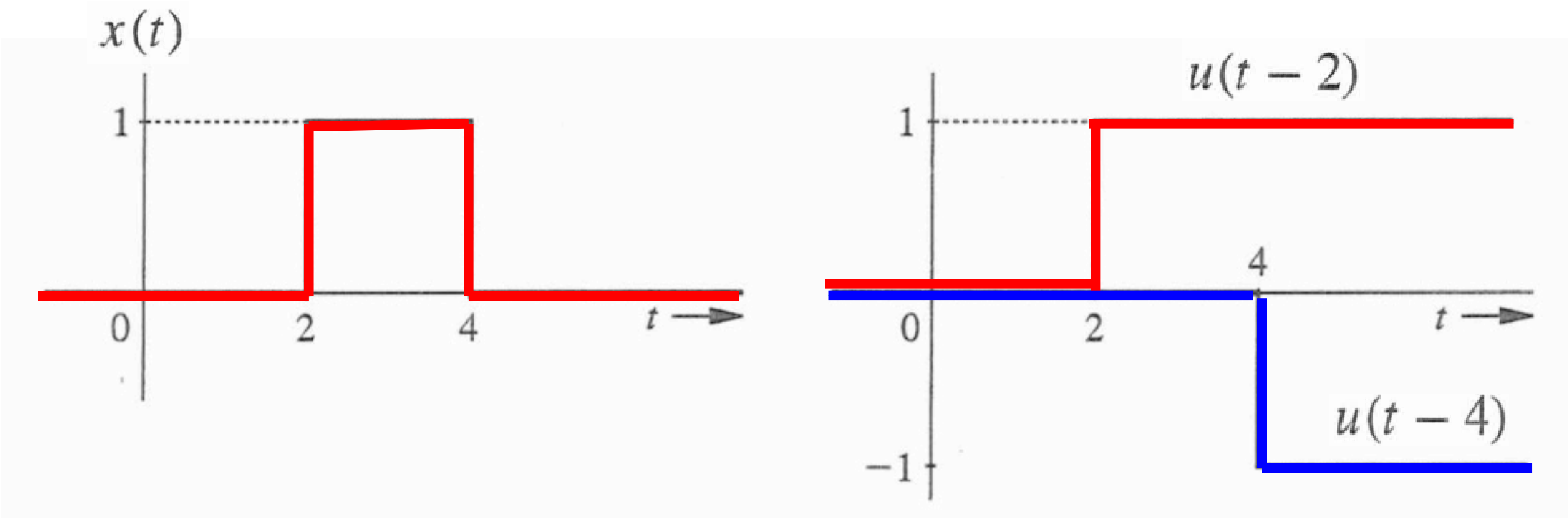
$$e^{-at}u(t)$$



Signal Models (2) – Pulse signal

- A pulse signal can be presented by two step functions:

$$x(t) = u(t-2) - u(t-4)$$

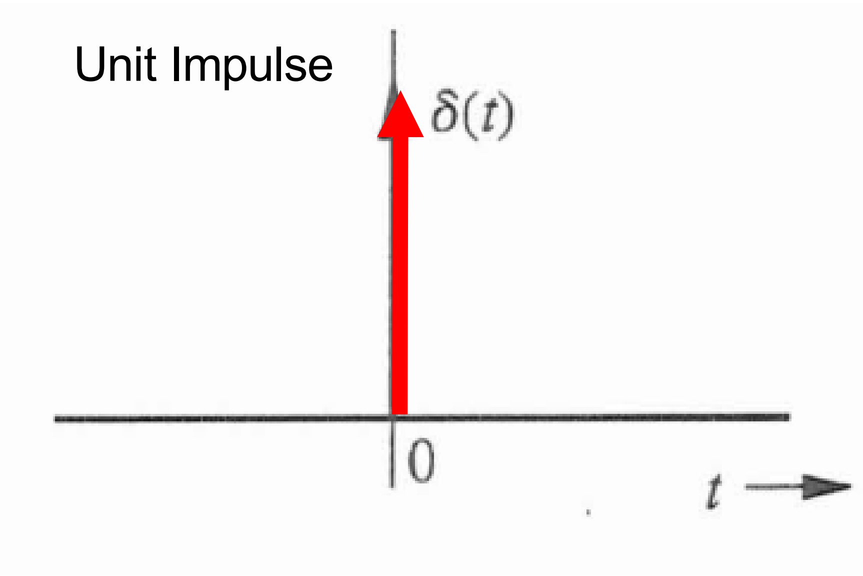


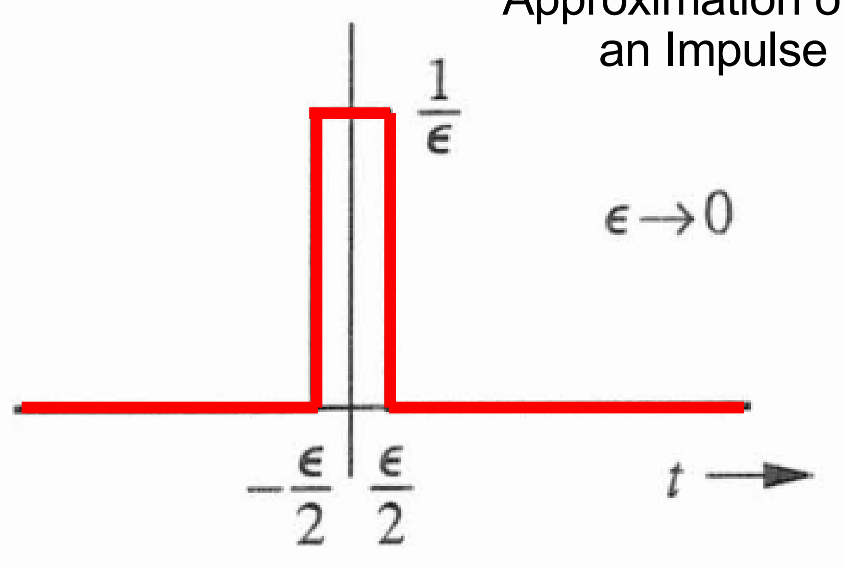
Signal Models (3) – Unit Impulse Function δ(t)

- First defined by Dirac as:

$$\delta(t) = 0 \qquad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$





Multiplying a function $\Phi(t)$ by an Impulse

- Since impulse is non-zero only at t = 0, and $\Phi(t)$ at t = 0 is $\Phi(0)$, we get:

$$\phi(t)\delta(t) = \phi(0)\delta(t)$$

- We can generalise this for t = T:

$$\phi(t)\delta(t-T) = \phi(T)\delta(t-T)$$

Sampling Property of Unit Impulse Function

- Since we have: $\phi(t)\delta(t) = \phi(0)\delta(t)$

- It follows that:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)dt = \phi(0)\int_{-\infty}^{\infty}\delta(t)dt$$
$$= \phi(0)$$

- This is the same as "**sampling**" $\phi(t)$ at t = 0.

- If we want to sample $\phi(t)$ at t = T, we just multiple $\phi(t)$ with $\delta(t - T)$

$$\int_{-\infty}^{\infty} \phi(t)\delta(t - T)dt = \phi(T)$$

- This is called the "**sampling property**" of the unit impulse.

The Exponential Function e^{st} (1)

- This exponential function is very important in signals & systems, and the parameter *s* is a complex variable given by:

$$s = \sigma + j\omega$$

- Therefore

$$e^{st} = e^{(\sigma+j\omega)t} = e^{\sigma t}e^{j\omega t} = e^{\sigma t}(cos\omega t + jsin\omega t) \quad \text{[eq 1]}$$

- Since $s^* = \sigma - j\omega$ (the conjugate of s), then

$$e^{s^*t} = e^{(\sigma-j\omega)t} = e^{\sigma t}e^{-j\omega t} = e^{\sigma t}(cos\omega t - jsin\omega t) \quad \text{[eq 2]}$$

- Eq 1 + Eq 2 gives:

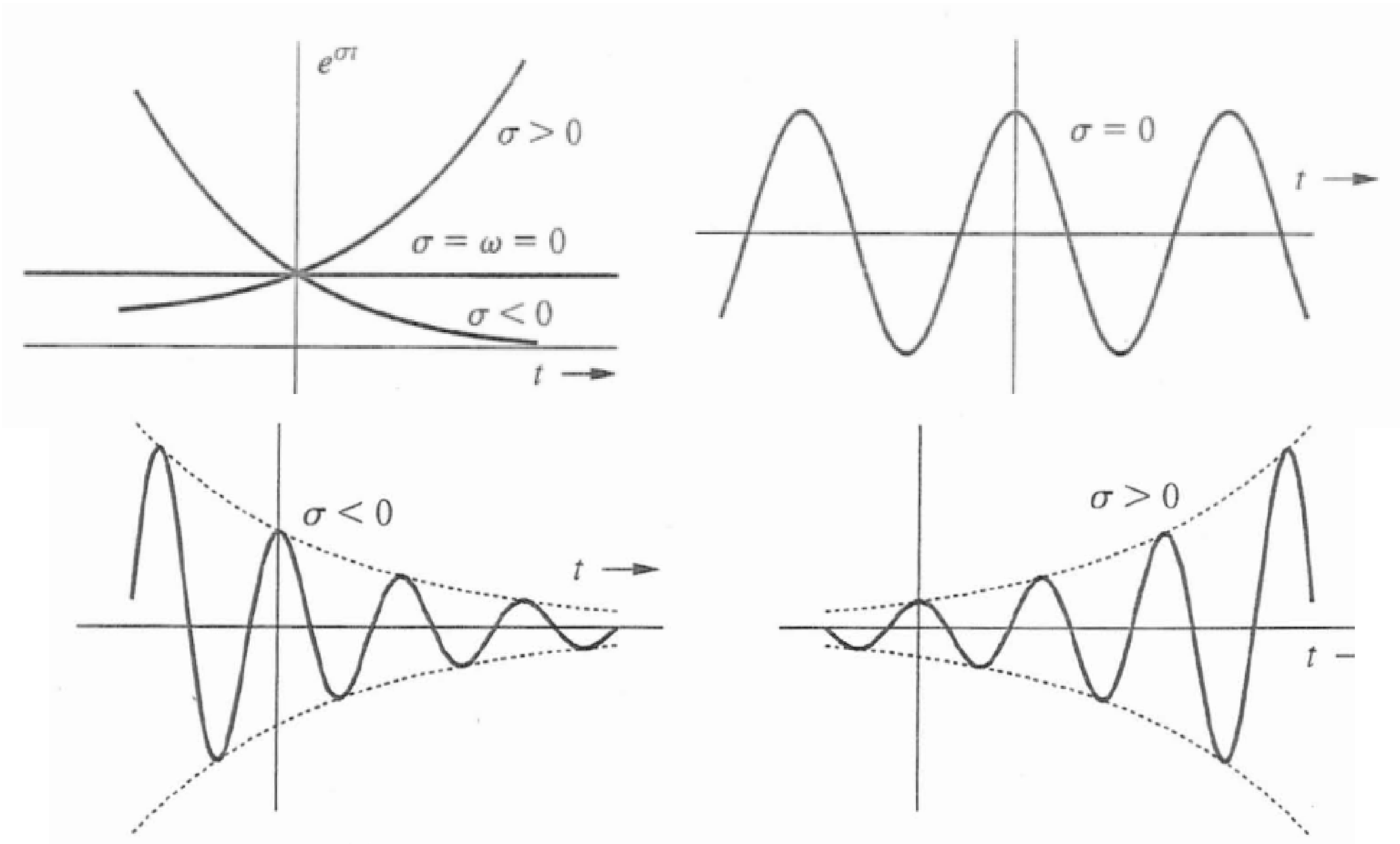
$$e^{\sigma t}cos\omega t = \frac{1}{2}(e^{st} + e^{s^*t})$$

The Exponential Function e^{st} (2)

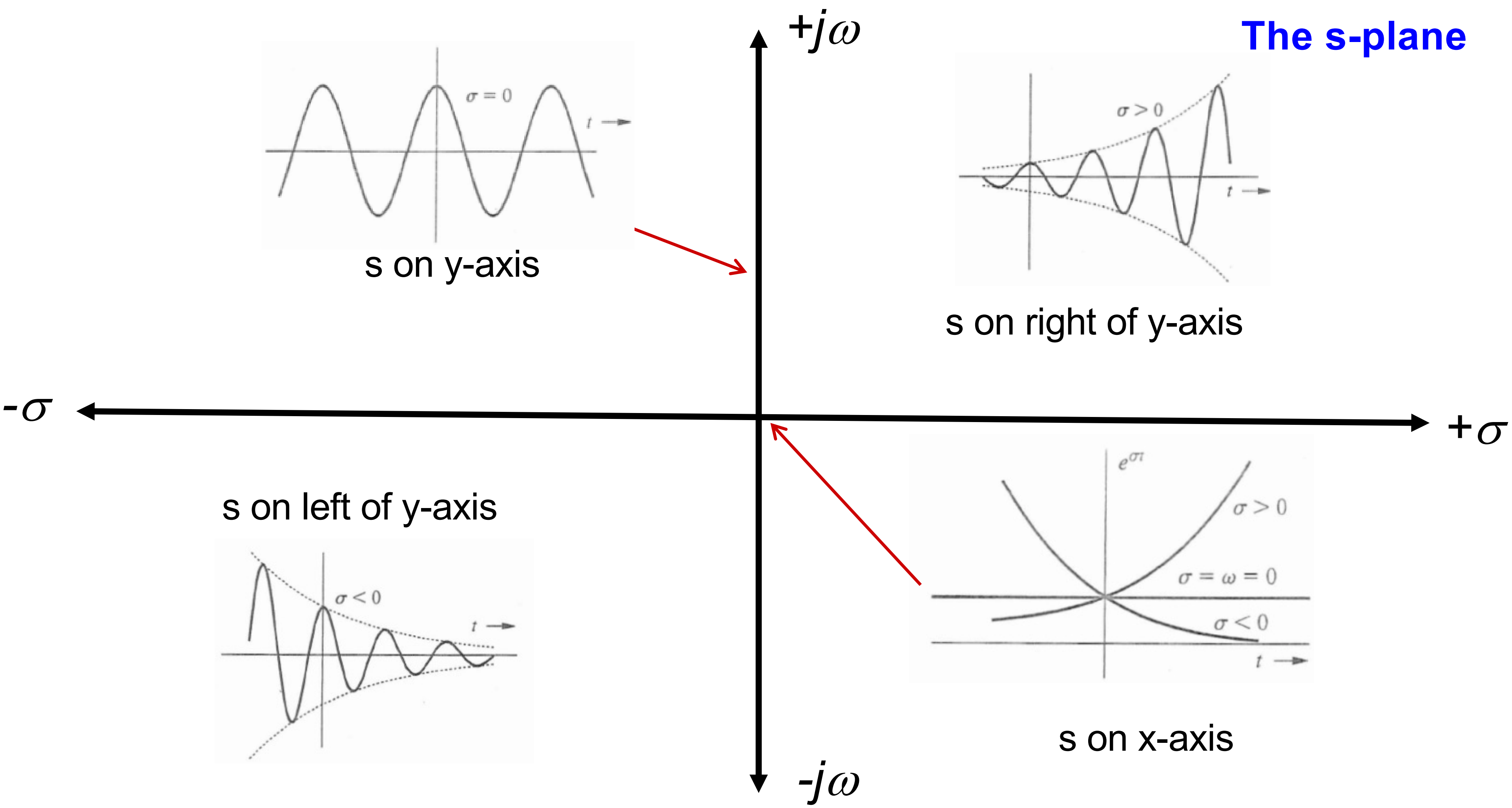
- If $\sigma = 0$, then we have the function $e^{j\omega t}$, which has a real frequency of ω
- Therefore the complex variable $s = \sigma + j\omega$ is the **complex frequency**
- The function e^{st} can be used to describe a very large class of signals and functions. Here are a number of example:

1. A constant $k = ke^{0t}$ $(s = 0)$
2. A monotonic exponential $e^{\sigma t}$ $(\omega = 0,\ s = \sigma)$
3. A sinusoid $\cos \omega t$ $(\sigma = 0,\ s = \pm j\omega)$
4. An exponentially varying sinusoid $e^{\sigma t} \cos \omega t$ $(s = \sigma \pm j\omega)$

The Exponential Function e^{st} (2)



The Complex Frequency Plane $s = \sigma + j\omega$



Three Big Ideas

1. The size of a time-limited signal is measured by it energy:

$$E_x = \int_{t_1}^{t_2} x^2(t)\, dt \qquad\qquad E_x = \sum_{n=1}^{N} x^2[n]$$

2. Delaying a signal x(t) by time T can be written as:

$$y(t) = x(t - T)$$

3. Unit impulse or delta function $\delta(t)$ can be use to model taking a sample from a signal. To take one sample of x(t) at time T is modelled as

$$x_T(t) = x(t) \times \delta(t - T)$$