

Lecture 2

Signals in Time Domain

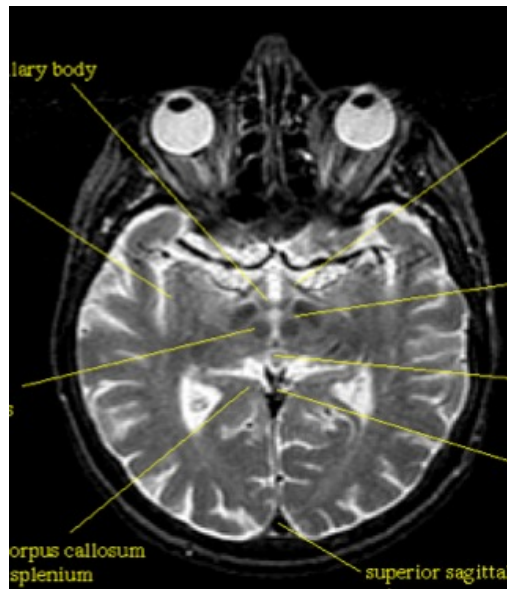
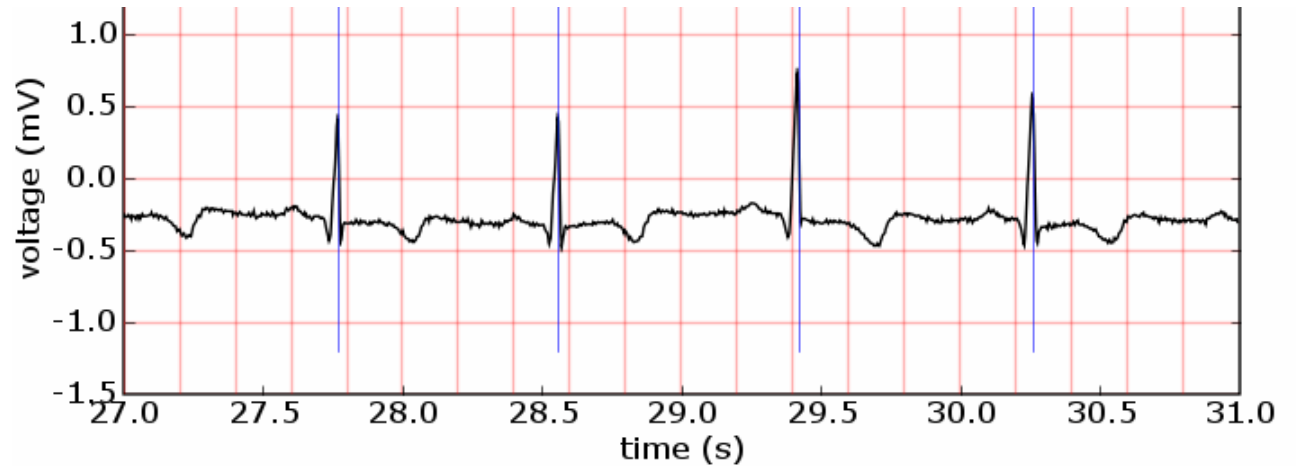
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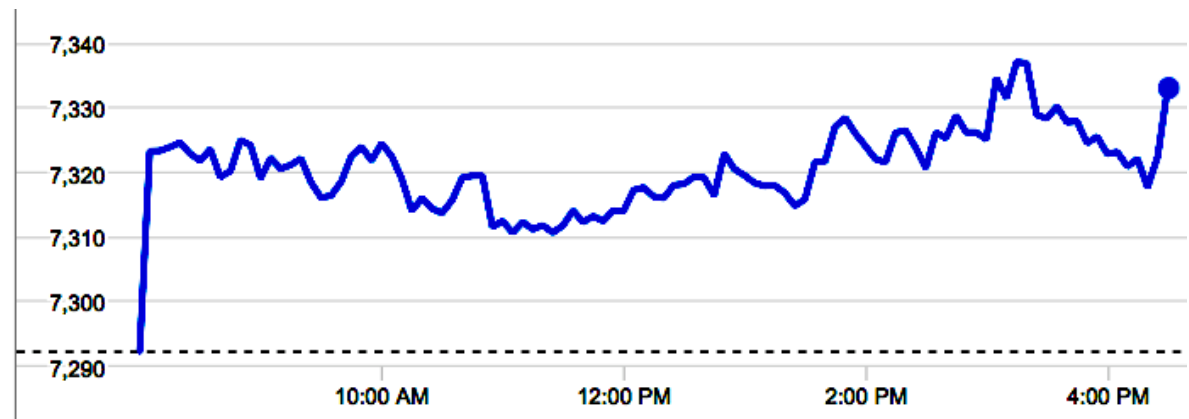
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Examples of signals

- ◆ Electrocardiogram (ECG) signal
- ◆ Magnetic Resonance Image (MRI) data as 2-dimensional signal



- ◆ FTSE 100 index in a day as signal (time series)

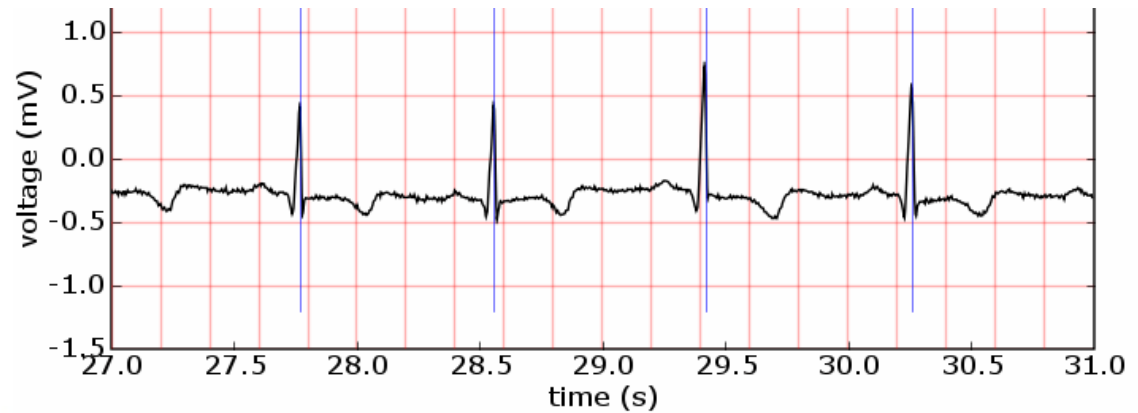


Signals Classification (1)

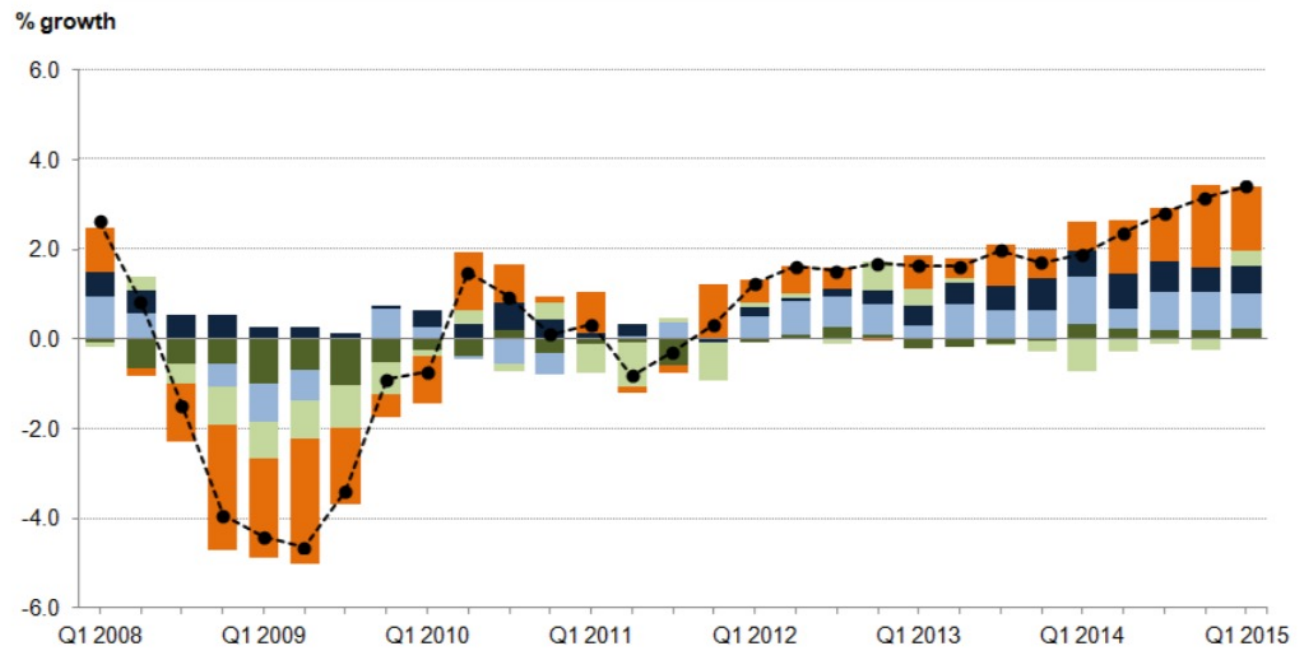
- ◆ Signals may be classified into:
 1. Continuous-time and discrete-time signals
 2. Analogue and digital signals
 3. Periodic and aperiodic signals
 4. Energy and power signals
 5. Deterministic and probabilistic signals
 6. Causal and non-causal
 7. Even and Odd signals

Signal Classification (2) – Continuous vs Discrete

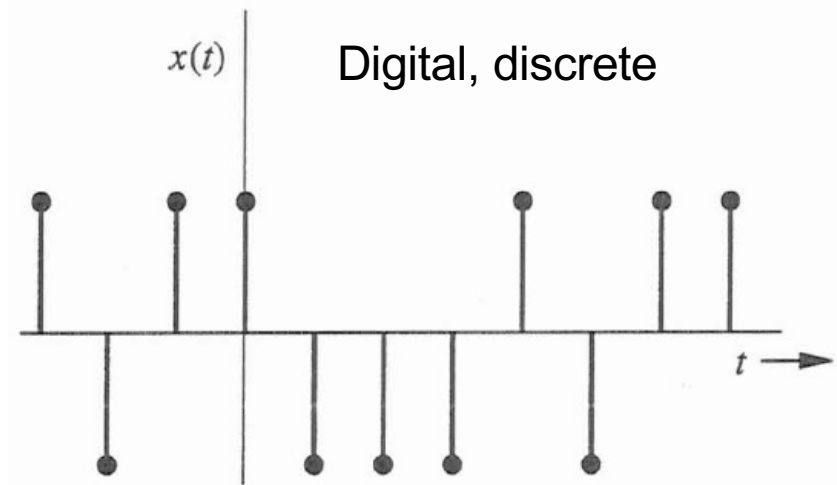
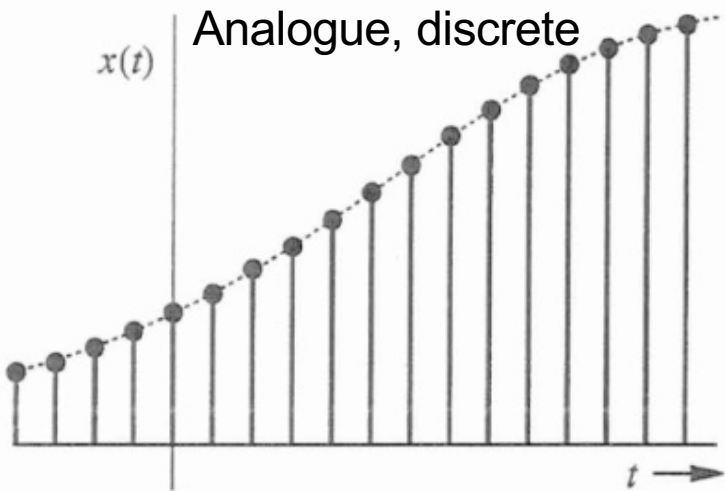
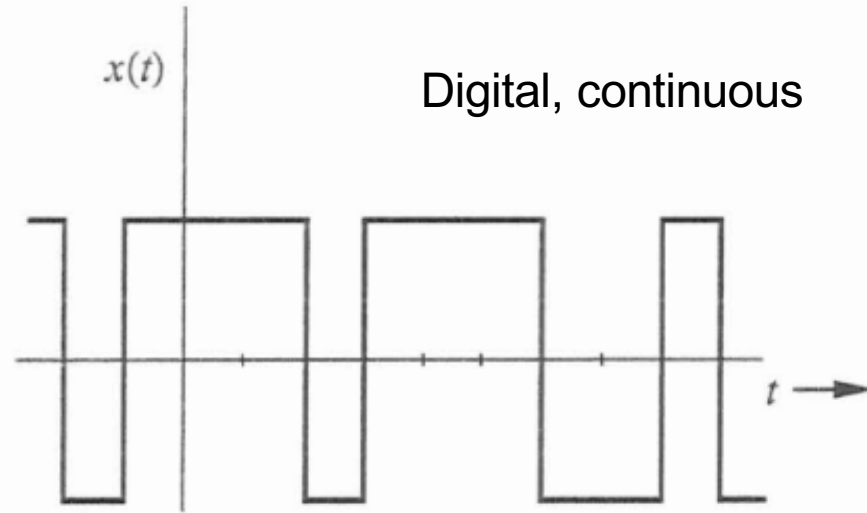
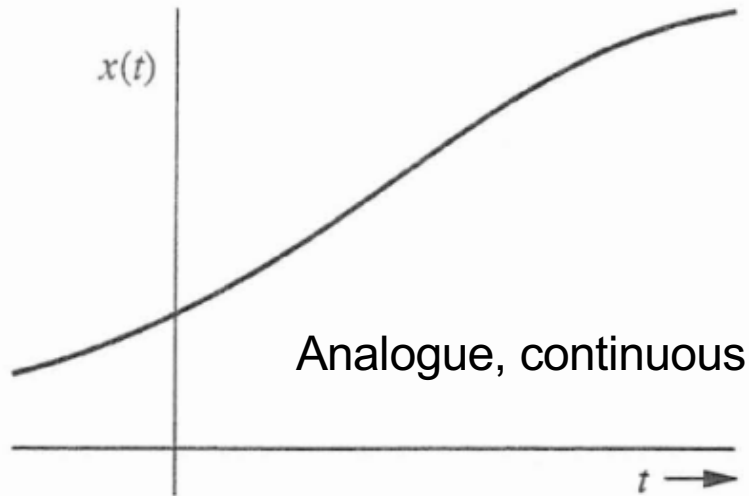
- ◆ Continuous-time (CT), e.g. ECG signal



- ◆ Discrete-time (DT), e.g. UK growth rate



Signal Classification (3) – Analogue vs Digital

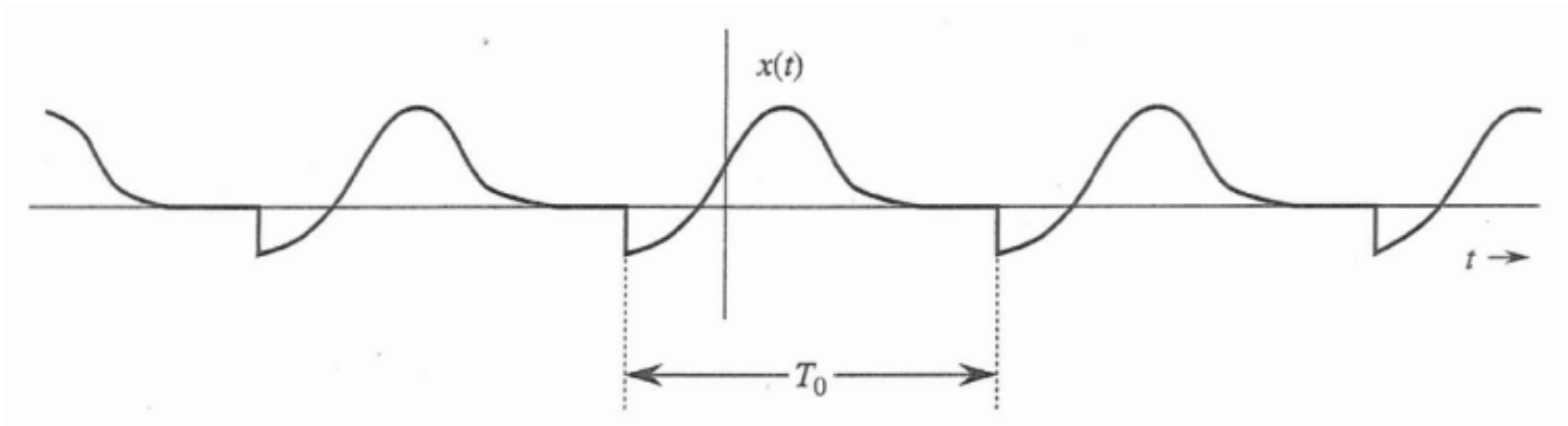


Signal Classification (4) – Periodic vs Aperiodic

- ◆ A signal $x(t)$ is said to be periodic if for some positive constant T_0

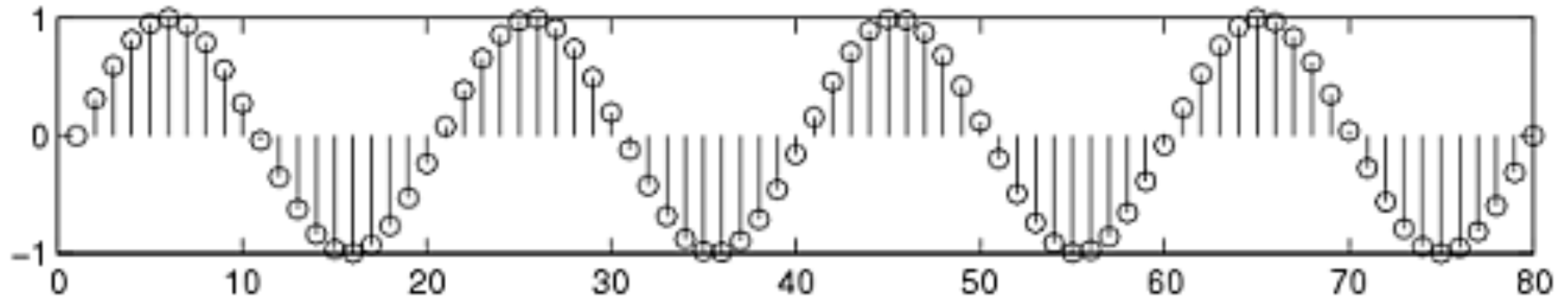
$$x(t) = x(t + T_0) \quad \text{for all } t$$

- ◆ The smallest value of T_0 that satisfies the periodicity condition of this equation is the *fundamental period* of $x(t)$.

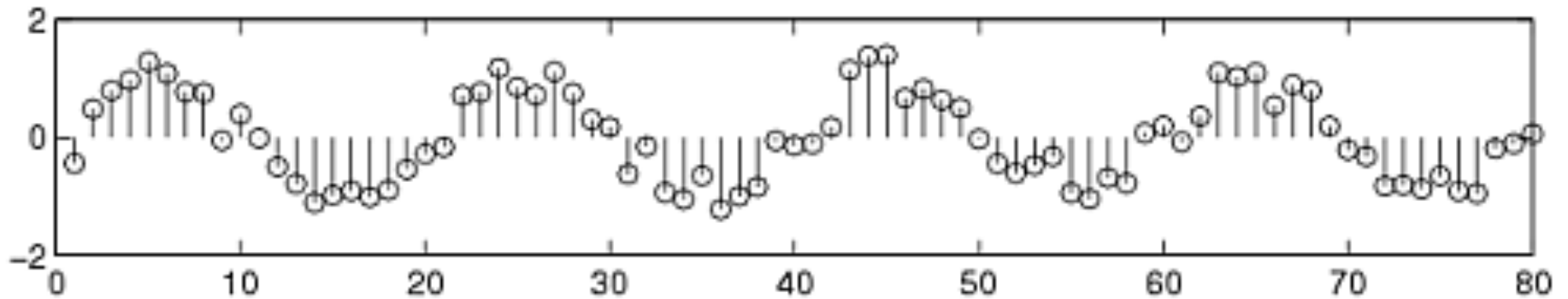


Signal Classification (5) – Deterministic vs Random

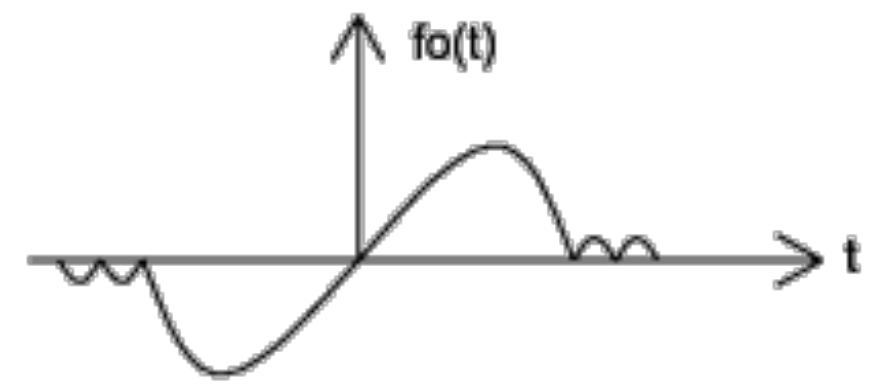
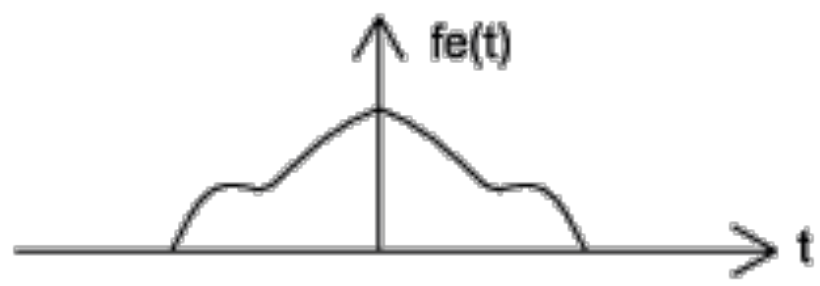
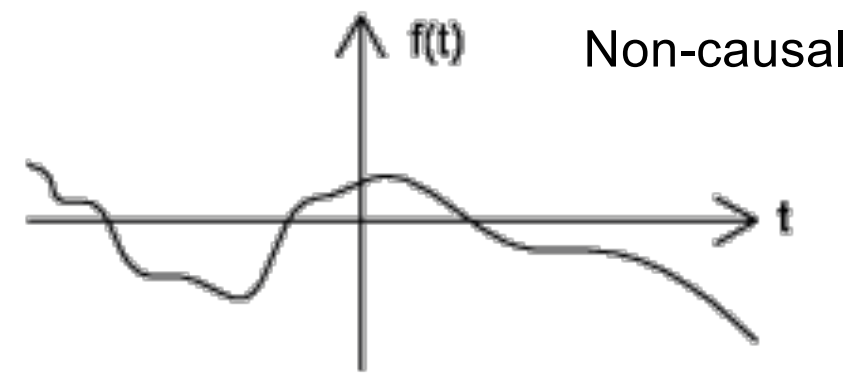
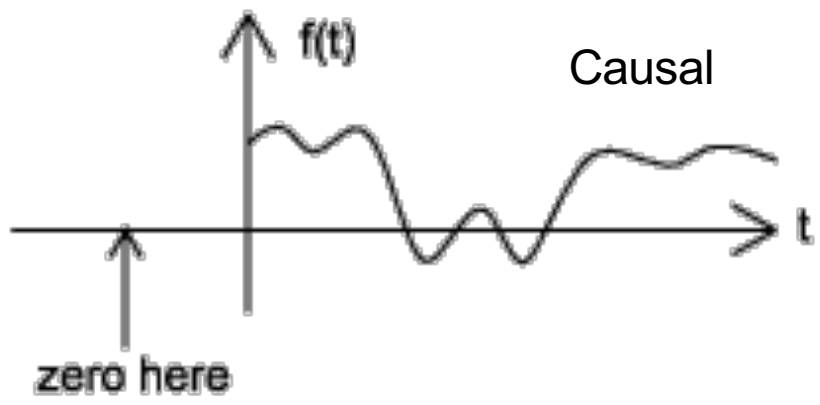
Deterministic



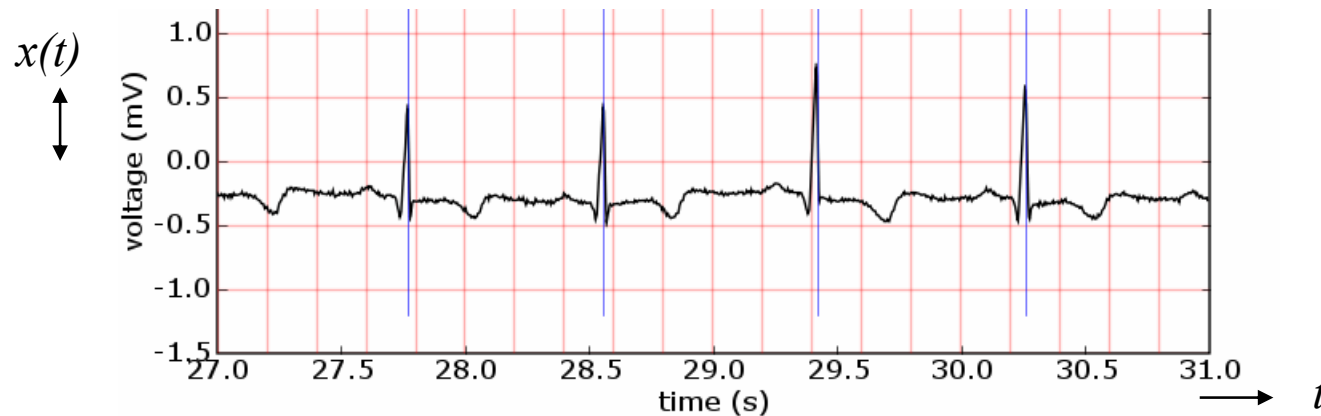
Random



Signal Classification (6) – Causal/Non-causal, Even/Odd



Size of a Signal $x(t)$ as energy



- ◆ Measured by signal energy E_x :

$$E_x = \int_{-\infty}^{\infty} x^2(t) dt$$

- ◆ Generalize for a complex valued signal to:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

- ◆ Energy must be finite, which means

$$\text{signal amplitude} \rightarrow 0 \quad \text{as} \quad |t| \rightarrow \infty$$

Size of a Signal $x(t)$ as power

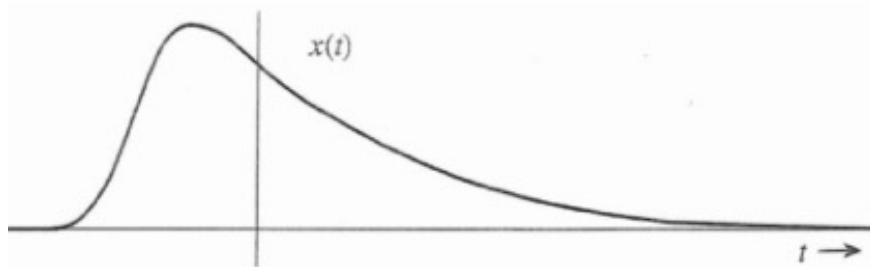
- ◆ If amplitude of $x(t)$ does not $\rightarrow 0$ when $t \rightarrow \infty$, need to measure power P_x instead:

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt$$

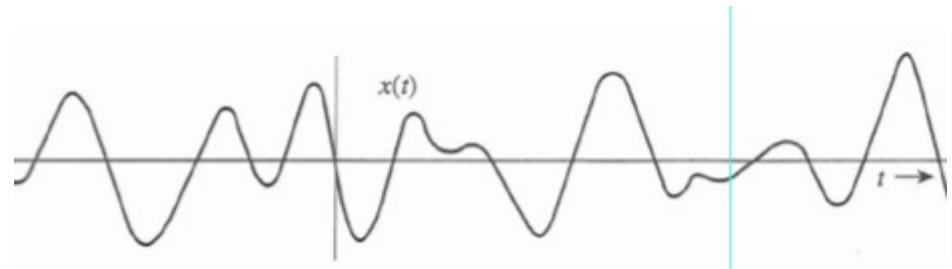
- ◆ Again, generalize for a complex valued signal to:

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

- ◆ Signal with finite energy
(zero power)



- ◆ Signal with finite power
(infinite energy)



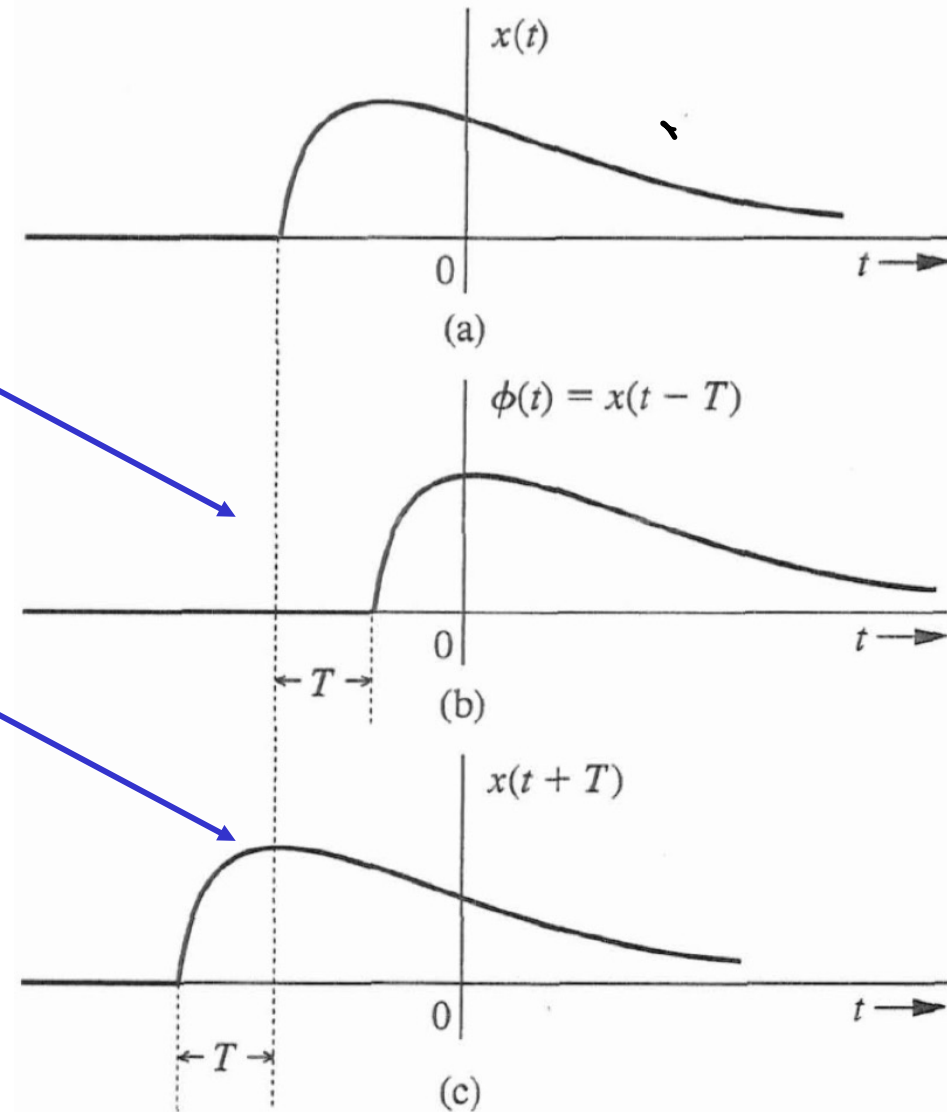
Useful Signal Operations –Time Shifting (1)

- ◆ Signal may be delayed by time T :

$$\phi(t + T) = x(t)$$

- ◆ or advanced by time T :

$$\phi(t - T) = x(t)$$



Useful Signal Operations –Time Scaling (2)

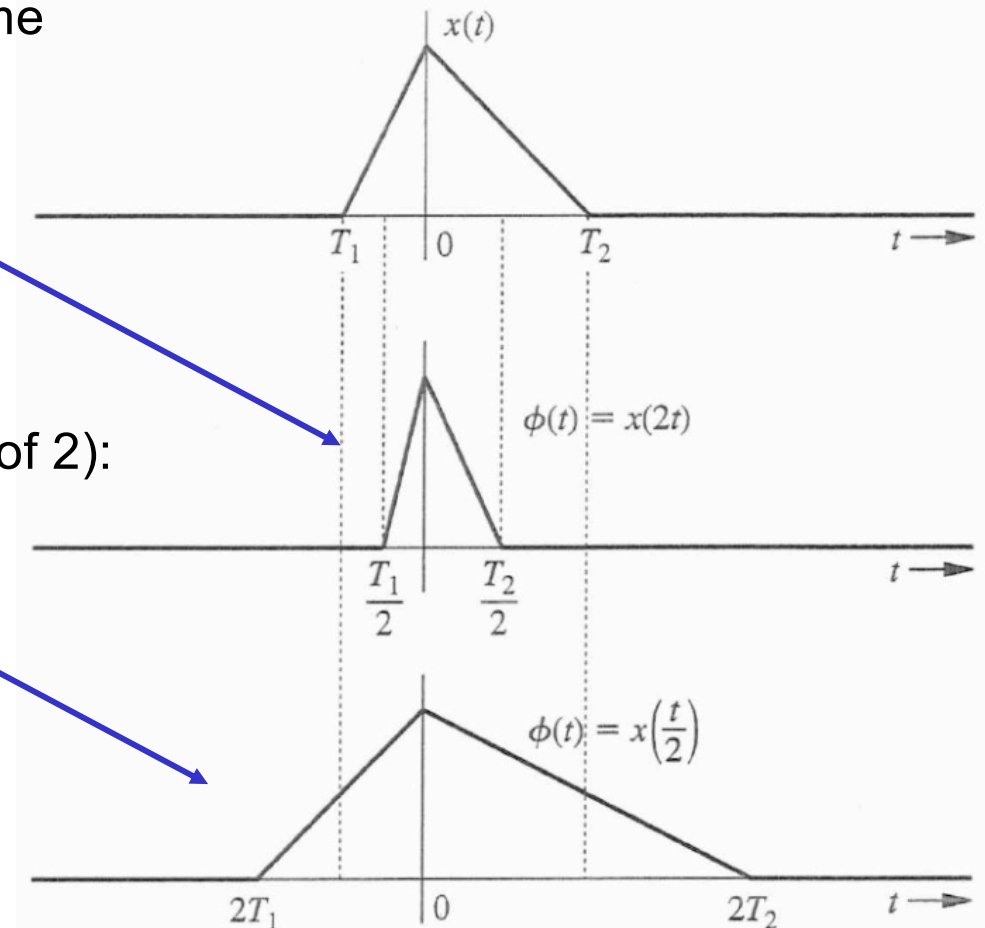
- ◆ Signal may be compressed in time (by a factor of 2):

$$\phi(t/2) = x(t)$$

- ◆ or expanded in time (by a factor of 2):

$$\phi(2t) = x(t)$$

- ◆ Same as recording played back at twice and half the speed respectively

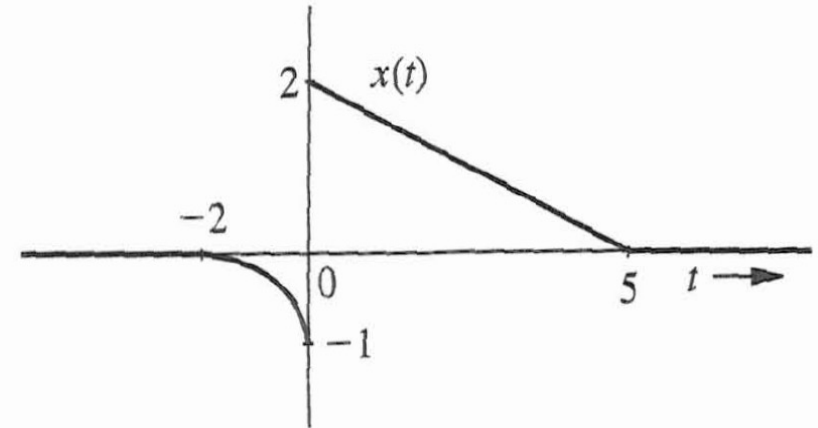


Useful Signal Operations –Time Reversal (3)

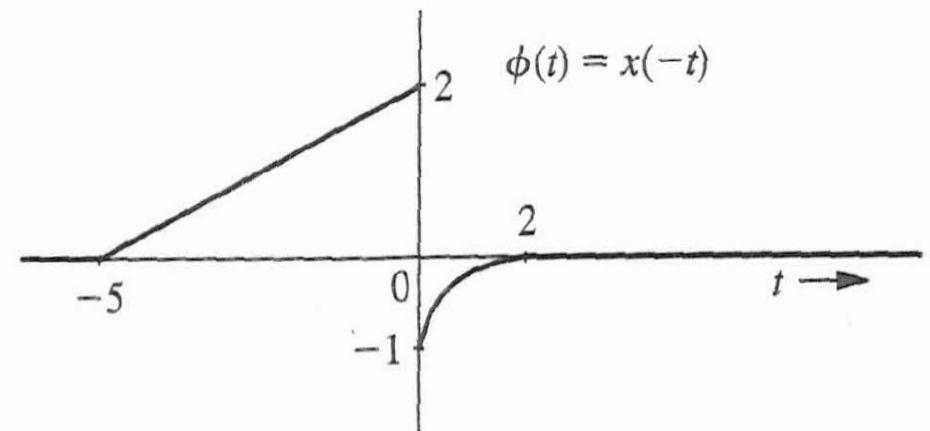
- ◆ Signal may be reflected about the vertical axis (i.e. time reversed):

$$\phi(t) = x(-t)$$

- ◆ We can combine these three operations.
- ◆ For example, the signal $x(2t - 6)$ can be obtained in two ways:
 1. Delay $x(t)$ by 6 to obtain $x(t - 6)$, and then time-compress this signal by factor 2 (replace t with $2t$) to obtain $x(2t - 6)$.
 2. Alternately, time-compress $x(t)$ by factor 2 to obtain $x(2t)$, then delay this signal by 3 (replace t with $t - 3$) to obtain $x(2t - 6)$.



(a)



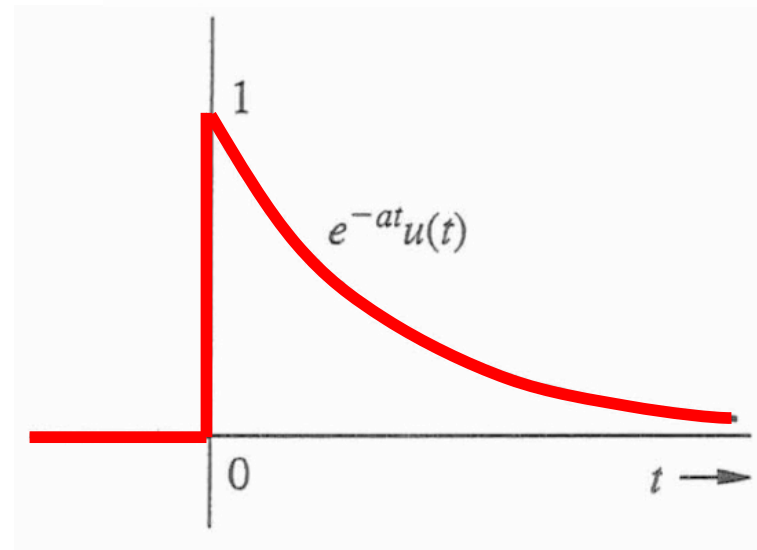
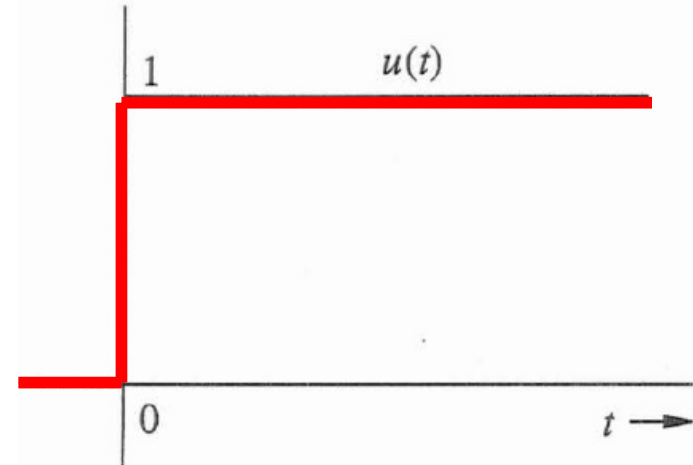
Signal Models (1) – Unit Step Function $u(t)$

- ◆ Step function defined by:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- ◆ Useful to describe a signal that begins at $t = 0$ (i.e. causal signal).
- ◆ For example, the signal e^{-at} represents an everlasting exponential that starts at $t = -\infty$.
- ◆ The causal form of this exponential can be described as:

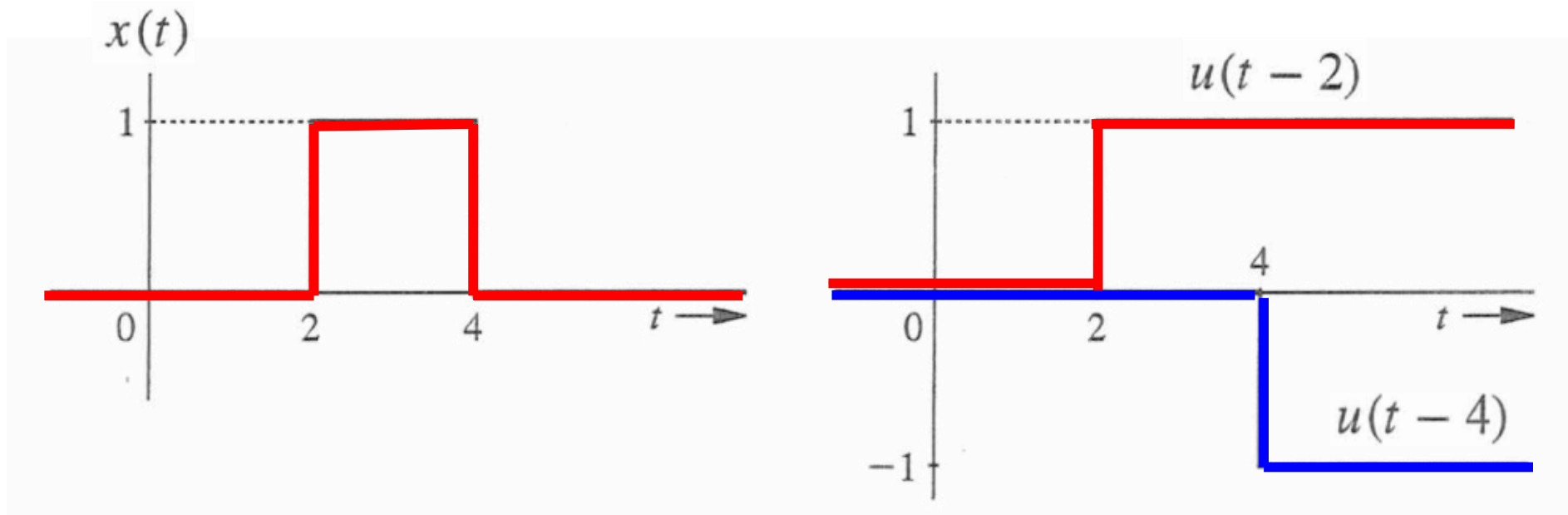
$$e^{-at}u(t)$$



Signal Models (2) – Pulse signal

- ◆ A pulse signal can be presented by two step functions:

$$x(t) = u(t - 2) - u(t - 4)$$

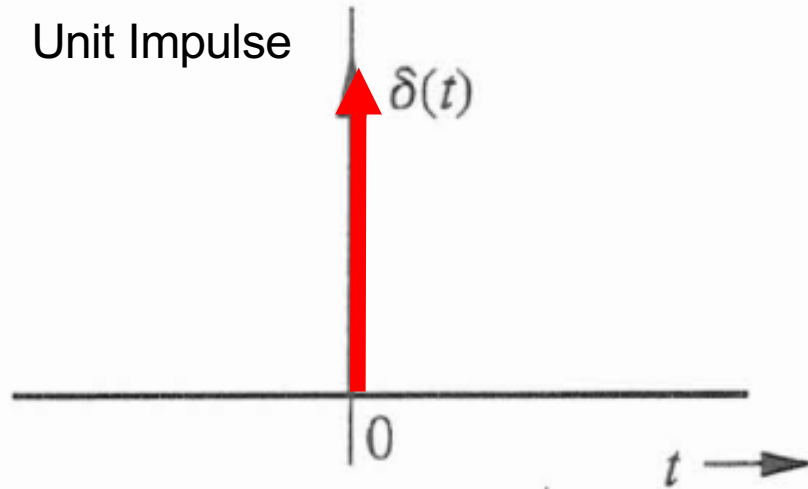


Signal Models (3) – Unit Impulse Function $\delta(t)$

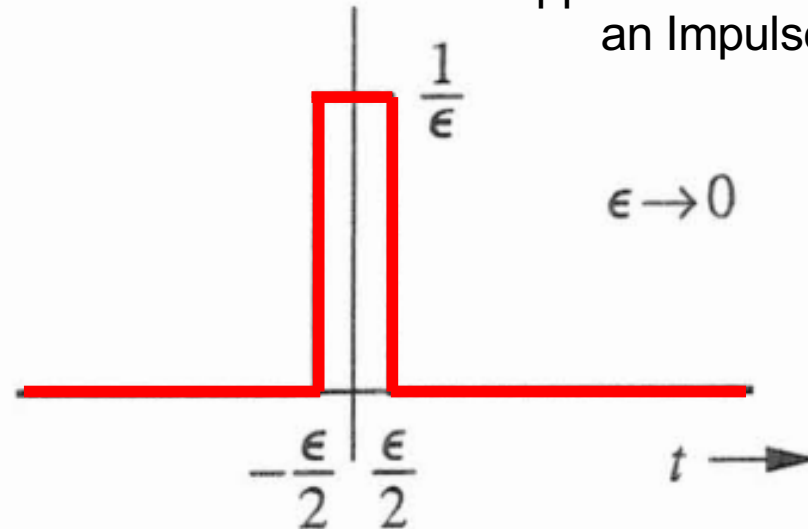
- ◆ First defined by Dirac as:

$$\delta(t) = 0 \quad t \neq 0$$
$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

Unit Impulse



Approximation of an Impulse



Multiplying a function $\Phi(t)$ by an Impulse

- ◆ Since impulse is non-zero only at $t = 0$, and $\Phi(t)$ at $t = 0$ is $\Phi(0)$, we get:

$$\phi(t)\delta(t) = \phi(0)\delta(t)$$

- ◆ We can generalise this for $t = T$:

$$\phi(t)\delta(t - T) = \phi(T)\delta(t - T)$$

Sampling Property of Unit Impulse Function

◆ Since we have: $\phi(t)\delta(t) = \phi(0)\delta(t)$

◆ It follows that:
$$\int_{-\infty}^{\infty} \phi(t)\delta(t)dt = \phi(0) \int_{-\infty}^{\infty} \delta(t)dt$$
$$= \phi(0)$$

◆ This is the same as “**sampling**” $\phi(t)$ at $t = 0$.

◆ If we want to sample $\phi(t)$ at $t = T$, we just multiple $\phi(t)$ with $\delta(t - T)$

$$\int_{-\infty}^{\infty} \phi(t)\delta(t - T)dt = \phi(T)$$

◆ This is called the “**sampling property**” of the unit impulse.

The Exponential Function e^{st} (1)

- ◆ This exponential function is very important in signals & systems, and the parameter s is a complex variable given by:

$$s = \sigma + j\omega$$

- ◆ Therefore

$$e^{st} = e^{(\sigma+j\omega)t} = e^{\sigma t} e^{j\omega t} = e^{\sigma t} (\cos\omega t + j\sin\omega t) \quad [\text{eq 1}]$$

- ◆ Since $s^* = \sigma - j\omega$ (the conjugate of s), then

$$e^{s^*t} = e^{(\sigma-j\omega)t} = e^{\sigma t} e^{-j\omega t} = e^{\sigma t} (\cos\omega t - j\sin\omega t) \quad [\text{eq 2}]$$

- ◆ Eq 1 + Eq 2 gives:

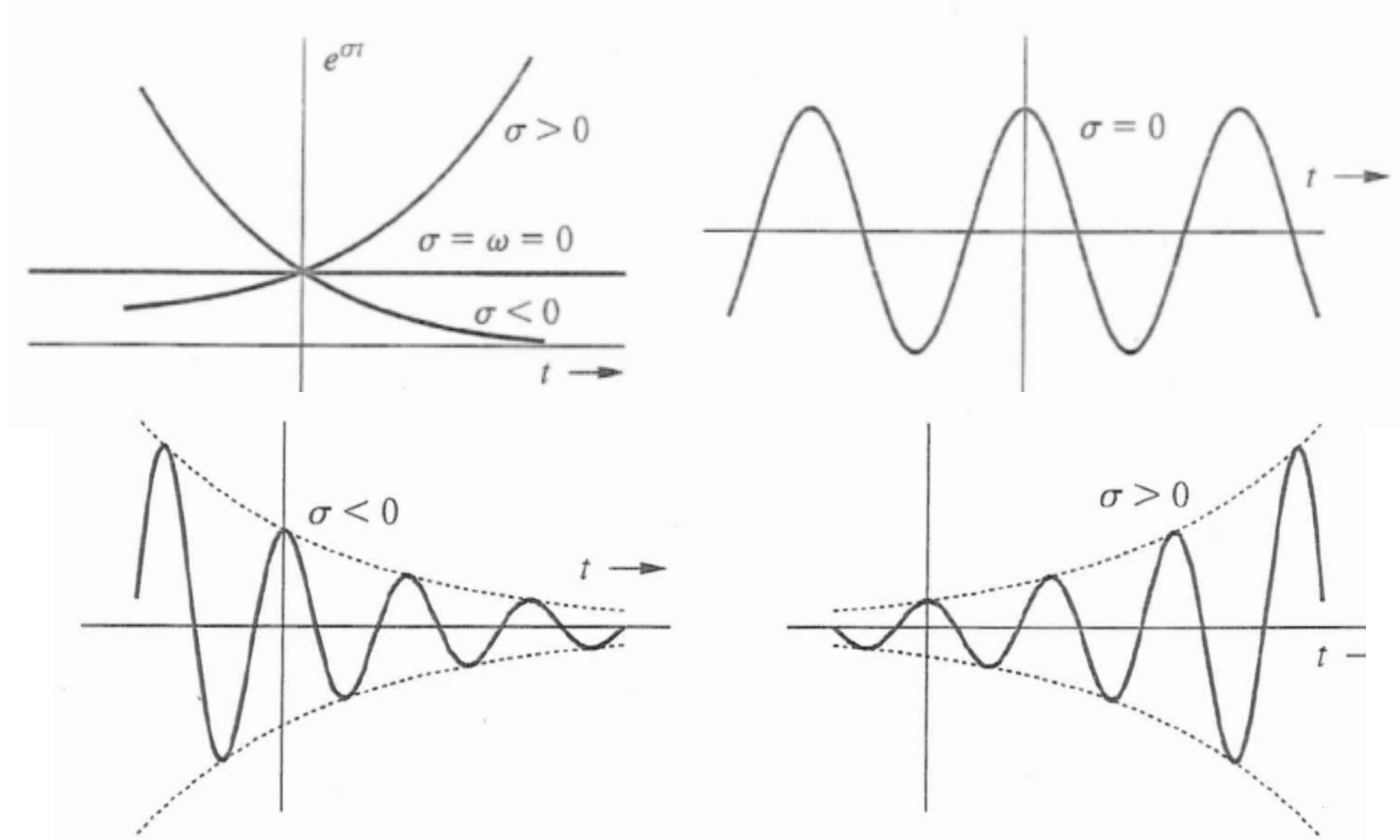
$$e^{\sigma t} \cos\omega t = \frac{1}{2} (e^{st} + e^{s^*t})$$

The Exponential Function e^{st} (2)

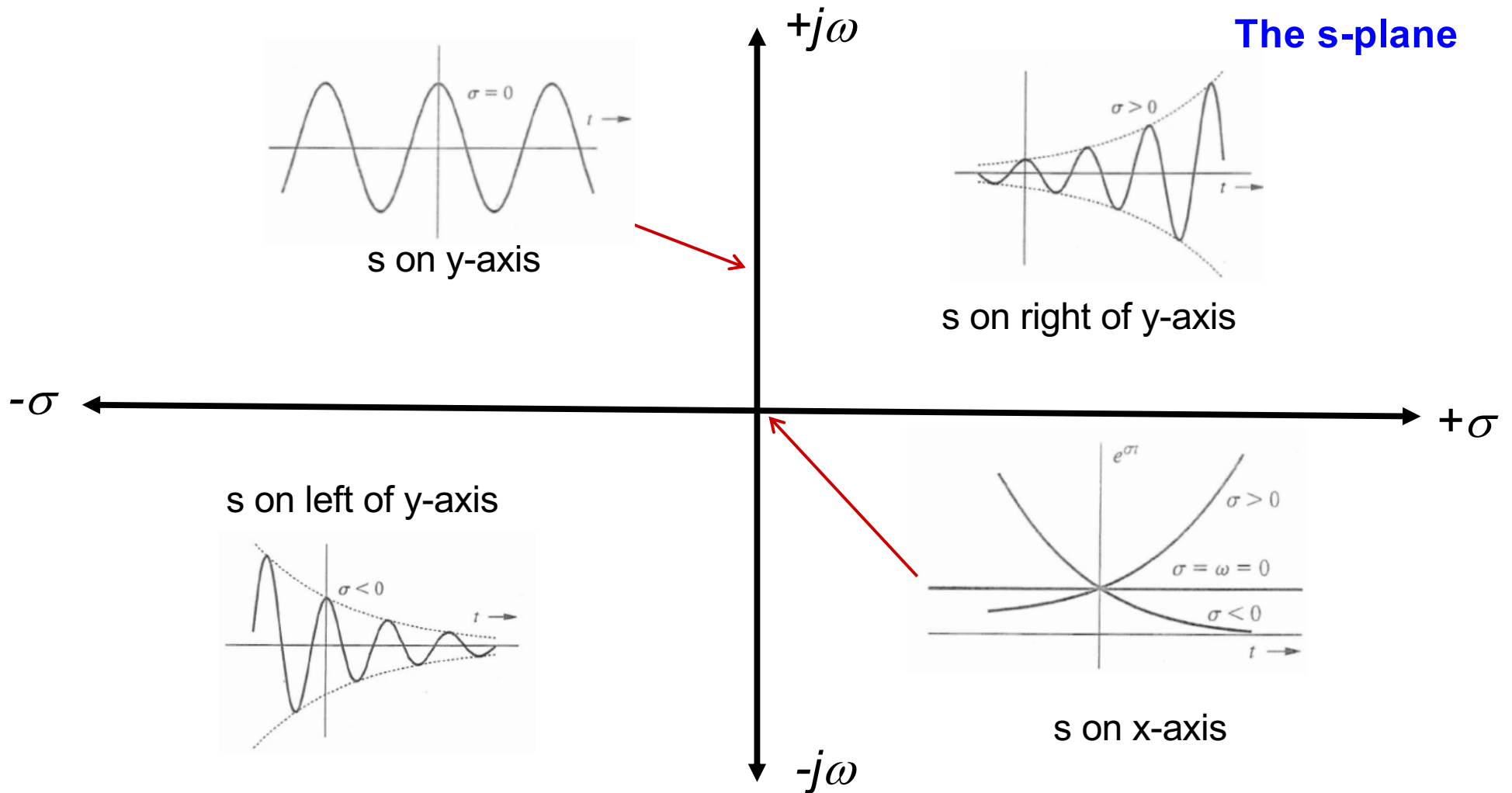
- ◆ If $\sigma = 0$, then we have the function $e^{j\omega t}$, which has a real frequency of ω
- ◆ Therefore the complex variable $s = \sigma + j\omega$ is the **complex frequency**
- ◆ The function e^{st} can be used to describe a very large class of signals and functions. Here are a number of example:

1. A constant $k = ke^{0t}$ ($s = 0$)
2. A monotonic exponential $e^{\sigma t}$ ($\omega = 0, s = \sigma$)
3. A sinusoid $\cos \omega t$ ($\sigma = 0, s = \pm j\omega$)
4. An exponentially varying sinusoid $e^{\sigma t} \cos \omega t$ ($s = \sigma \pm j\omega$)

The Exponential Function e^{st} (2)



The Complex Frequency Plane $s = \sigma + j\omega$



Three Big Ideas (1)

1. The size of a time-limited signal is measured by its energy:

$$E_x = \int_{t_1}^{t_2} x^2(t) dt \qquad E_x = \sum_{n=1}^N x^2[n]$$

2. Delaying a signal $x(t)$ by time T can be written as:

$$y(t) = x(t - T)$$

Three Big Ideas (2)

3. Unit impulse or delta function $\delta(t)$ can be used to model taking a sample from a signal. To take one sample of $x(t)$ at time T is modelled as

$$x_T(t) = x(t) \times \delta(t - T)$$

